Stability of Cartels in Multimarket Cournot Oligopolies*

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Abstract

We investigate the stability of cooperation agreements, such as those agreed by cartels, among firms in a Cournot model of oligopolistic competition embedded in a multimarket contact setting. Our analysis considers a broad array of 64 potential market structural configurations under linear demand and quadratic production costs. We establish that for an appropriate range of parameter values there exists a unique core stable market configuration in which an identical two-firm cartel is sustained in *both* markets. Our result highlights the significance of multimarket presence for cartel formation in light of the well-known result from the single-market setting where cartels are non-profitable.

1 Introduction

In their seminal work, Salant, Switzer and Reynolds (1983) make the observation that total profits of firms are likely to be higher when they act as individual profit-maximizers than when they choose their actions jointly as part of a cooperation agreement. This led to the coining of the term *merger paradox* to describe this fundamental insight in the setting of Cournot competition in a single market.

There are numerous implications of this fundamental insight for the sustainability of informal (cartel) and formal (merger) agreements among firms in a single market. In essence the paradox seeks an answer to the question why firms agree to cooperate when they are better-off acting on their own. Various authors have since investigated how this result depends on the modelling assumptions concerning the functional form of demand and production cost. With sufficiently convex costs, for example, Perry and Porter (1985) and Amir and Stepanova (2009) show that the merger paradox disappears.

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Other authors have demonstrated the sensitivity of the result on the precise order or sequencing of decisions and the presence of outside competitive pressure. Following an approach pioneered by d'Aspremont, Jacquemin, Gabszewicz and Weymark (1983) for example, Shaffer (1995), Konishi and Lin (1999) and Zu, Zhang and Wang (2012) demonstrate the existence of a stable cartel where the cartel is a Stackelberg quantity leader and all non-member firms of the cartel are Cournot competitive with respect to the residual demand.

Instead, we focus on environments in which firms operate on multiple, strongly related markets—denoted as *multimarket oligopolies*. In these multimarket oligopolies we investigate the endogenous composition and location of cartel structures. The setting of multiple markets on which these firms operate can be interpreted as their presence at multiple geographical locations or as representing the separation between different media through which trade is conducted. The latter might refer to a traditional sales technology through stores versus virtual sales through online web stores. We make a contribution to this literature by demonstrating that stable agreements are sustained in a standard Cournot oligopoly if firms operate on multiple (strongly related) markets and that these agreements involve a strict subset of firms operating on those markets.

For this insight we do not rely on the agreeing cartel coalition having a quantity-setting leadership position. We apply convex production costs, creating both strategic substitutes and diseconomies of scope across both markets—using the terminology of Bulow, Geanakoplos and Klemperer (1985).[1] Our model is therefore related to that of Zhang and Zhang (1996). These authors provide conditions for the existence of Cournot-Nash equilibrium in multimarket environments. However, they do not consider the possibility of cooperative agreements among market participants. Such cartel formation is instead the focus of our work.

A key element in any analysis of cartel formation is what constitutes a sustainable cooperation agreement leading to a stable cartel. For this purpose we choose to employ the notion of *core stability* as it best captures the range of deviation possibilities of the firms—unilaterally or as a group; and the concept of self-enforcing, binding agreements. A core stable configuration is one in which there are no incentives for any group of firms to deviate, either by forming an alternative cartel or individually. We build our notion on the premise that the deviating firms would expect the non-deviating firms to maintain the status-quo, i.e., by definition the non-deviating firms are assumed to be passive and, therefore, the market structures involving non-deviating players are not expected to change.

Related literature. Belleflamme and Bloch (2008) study conditions for sustainable cooperation between two firms in a symmetric two-market setting. Our work differs from theirs in that we allow for cartel formation among *three* firms in possibly asymmetric markets, i.e., cartels may be formed by fewer than all market participants and the two markets may

[1] Assuming linear costs in a multi-market environment does not add to the analysis of cartel stability since there are no linkages across markets, so the multimarket oligopoly is simply the original single-market oligopoly replicated twice independently.

differ in size. This allows us to provide further insights on the degree of cooperation and the location of the cartel based on the relative market size.

While multimarket oligopolies have been used to analyse corporate espionage (Billand, Bravard, Chakrabarti and Sarangi, 2016) and taxation (Lapan and Hennessy, 2011), we believe we are the first to examine cartel stability in multi-market oligopolies.

We note here that our underlying cartel formation game is akin to a partition function form game—a model that has been studied in cooperative game theory. In a partition function form game—as in our cartel formation setup—the value a coalition of players can generate in cooperation depends on the structure of cooperative agreements among the players outside of this coalition. Our multimarket cartel formation model, however, is more general than a partition function form game as players operate on multiple distinctive market settings: each firm may be part of multiple agreements—one on each market; the structures of cooperative agreements on each market may differ; and all market structures affect the payoffs of all cartels and all individual firms.

Concerning the fundamental notion of core stability as pursued here, we note that this notion originates in the theory of cooperative games in partition function form, seminally, by Chander and Tulkens (1997). More recently, Abe and Funaki (2017) examine the optimistic and pessimistic core of such games. In the pessimistic core, the deviating coalition assumes the worst reaction from the remaining non-deviating players. In the optimistic core, it assumes the best reaction from the non-deviating players. They also consider notions of the core where the non-deviating players are expected to dissolve their coalitions into singletons or form the largest possible coalition. This approach is not directly applicable to our setup, however, as in our model profits among cartel members are non-transferable.

As an alternative approach to the one we have taken—build on a standard cooperative game theoretic concept— one could consider a non-cooperative game theoretic approach to coalition formation. We mention a few papers that have taken this approach. A comprehensive review of this literature is available in Yi (2003).

Bloch (1996) puts forward the following non-cooperative procedure. Based on an exogenous order, players propose coalitions which other coalition members can accept or reject. The first player to reject the offer makes a counter-offer in the next period and so forth. If all players agree, the coalition is formed and the coalition is not allowed to subsequently break-up or accept new members, and the remaining players continue the coalition formation process. Bloch (1997, 2002) shows that the equilibrium coalition structure corresponds to the equilibrium of a certain size announcement game and involves a no-delay equilibrium in which all proposals are accepted. In the homogeneous Cournot model, this procedure results in the formation of a single cartel.

Yi and Shin (2000) and Bloch (2002) consider a different procedure where players simultaneously decide whether or not to join the coalition: Each player announces an address simultaneously, and players with the same address belong to the same coalition.

Ray and Vohra (1997) analyse a coalition formation game in which coalitions can only break up into smaller sub-coalitions. They define stable coalition structures as follows: A degenerate coalition structure in which all coalitions are singletons is stable by definition. A non-degenerate coalition structure is stable if no coalition has a sub-coalition—called a coalition of leading perpetrators—which members have incentives to initiate a break-up of the coalition structure. Players are far-sighted and foresee the final stable coalition structure that will form after further subsequent break-ups. The leading perpetrators will only initiate a break-up if they are better off in the final stable coalition structure.

Ray and Vohra (1999) define an extensive form bargaining game similar to Bloch (1996) that yields such a stable coalition structure in the subgame perfect stationary equilibrium. One difference with Bloch (1996) is that while in the latter, coalitional worth is divided according to a fixed rule, in the former, how the worth is divided is part of the bargaining process. In the homogeneous Cournot model, once again a unique cartel emerges.

The advantage of the approach we take compared over the non-cooperative game theoretic analysis, is in its generality: Unlike non-cooperative concepts, the core is not tied to a specific procedure for the sharing of the coalitional value or pre-determined sequence of deviations.

In addition, our analysis is built on the premises on non-transferability of payoffs. In this respect, our work is related to the literature on hedonic coalition formation games. These game theoretic models were introduced by Bogomolnaia and Jackson (2002), further analysed by Banerjee, Konishi and Sonmez (2001). In this setting, the players' preferences are ordinal and they are defined over coalition memberships. Various notions of stability have been defined in this setting. Individual stability refers to the situation where no player wants to leave her coalition for another one (including the empty coalition). Nash stability is a stronger version of individual stability where players can join any new coalition without the permission of existing members. More demanding than these is core stability, where multiple players can deviate to form a new coalition. While in hedonic games, a player's payoff depends only on the membership of her coalition, the approach can be extended to non-hedonic games where a player has a preference mapping over all possible partitions of the player set. Core stability can be extended to this setting which is what we are doing here.

Structure of the paper: The next section presents the multimarket Cournot model and key concepts for its analysis. In Section 3 we present the main results on core stable structures and discuss the role of convexity. Section 4 contains a brief discussion of other assumptions and outlays paths for future work.

2 The Model

We explore a setting where three firms are selling an identical product in multiple, separate markets. The firms are labelled a, b and c and we denote the set of firms by $N = \{a, b, c\}$ with i being a generic firm in the set N. We let the set of markets be given by $\mathcal{M} = \{M_1, M_2, \dots, M_m\}$, using the indicator $k \in \{1, \dots, m\}$ to refer to market $M_k \in \mathcal{M}$.

We assume that all firms in N are quantity-setters; that is, in the absence of cooperation agreements these firms compete á la Cournot in quantities on all markets in $\mathcal{M}$. We denote by q_{ik}, the quantities sold by firm i in market M_k. Furthermore, we let p_k stand for the market price emerging on market M_k.

We assume that competitors' products are substitutes in all markets and there are diseconomies of scope across these markets. More specifically, markets are characterised by inverse linear demand functions specified for M_k as

$$p_k = \alpha_k - \sum_{i \in N} q_{ik} = \alpha_k - q_{ak} - q_{bk} - q_{ck},$$

where $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_m$ are demand parameters capturing each market's size, respectively. Hence, all markets are ordered in terms of their size, where market M_1 is the smallest market and market M_m is the largest market.

We assume that firms produce under an identical quadratic cost function given by

$$C(q_i) = \frac{1}{2}\left(\sum_{k=1}^{m} q_{ik}\right)^2$$

where $q_i = (q_{i1}, q_{i2}, \ldots, q_{im})$ is the output vector for firm $i \in N$.

We remark that from the postulated linearity of the market demand, it follows that the total revenues of a firm are simply the sum of the firm's revenues in each market. On the other hand, there is no such market separability in the postulated quadratic cost function. This implies that firm i's total profit can be expressed as

$$\begin{aligned} \pi_i(q_i) &= \sum_{k=1}^{m} p_k \, q_{ik} \, - C(q_i) \\ &= \sum_{k=1}^{m} (\, \alpha_k - q_{ak} - q_{bk} - q_{ck} \,) \, q_{ik} - \frac{1}{2}\left(\sum_{k=1}^{m} q_{ik}\right)^2 \end{aligned} \tag{1}$$

For this linear-quadratic formulation, the second order conditions for a maximum are always satisfied if α_k's are not too different.[2] Hence, one obtains a unique interior maximum through consideration of the first order conditions.

2.1 Cartel Formation

We consider a general cooperation framework in which *any subset* of the postulated three firms may choose to form a cartel *in any* market. Thus, a *cartel* is any coalition of at least two firms in any one of the m markets in $\mathcal{M}$.

Note that in principle any given cartel operates in a single market, but we emphasise that the same group of firms may form a cartel in other markets as well.[3] Equally, there may be distinct groups that form cartels on different markets. Thus, whereas each firm can be a member of only one cartel in a given market[4], the same firm may be a member of more than one cartel, each of which operates in a different market and has a distinct membership.

[2] For instance, if $k = 2$, we require $\alpha_1 > \frac{1}{5}\alpha_2$, otherwise all firms will produce zero output in market M_1.

[3] We re-visit our modelling strategy of a cartel as being bound to a single market in the discussion section.

[4] Given that our analysis pertains to a set of three firms, the assumption implies that there is at most one cartel operating in a market.

Such cartel formation is formalised as follows.

Definition 1 *A* ***market configuration*** *is a listing of individual market structures* $\mathbf{\Omega} = \langle \omega_1, \dots, \omega_m \rangle$ *where for each market* $M_k \in \mathcal{M}$*, the structure* ω_k *is a partition of* $N = \{a, b, c\}$ *into distinct groups.*
For every firm $i \in N$*, we denote by* $i(\omega_k)$ *the status of firm* i *in the structure* ω_k *in market* M_k *given by the group* $i(\omega_k) = S \in \omega_k$ *if* $i \in S$ *.*

A market structure imposes on each market a partitioning of all firms that describes the competitive structure in that particular market. For example, consider $m = 2$ and the following market structure: $\omega_1 = \{\,\{a\}, \{b\}, \{c\}\,\}$, i.e., pure competition in market M_1; and $\omega_2 = \{\,\{a\}, \{b, c\}\,\}$, i.e, partial cooperation—a cartel between firms b and c in market M_2. In this case we have firm a acting as singleton in both markets with $a(\omega_1) = a(\omega_2) = \{a\}$, whereas firms b and c have a different status in each market: $i(\omega_1) = \{i\}$ and $i(\omega_2) = \{ij\}$ with $i, j \in \{b, c\}$ and $i \neq j$.

Cartel objectives. The objective of a cartel operating on a specific market is in principle to maximize the joint profits of its members. Thus, members of the cartel commit to an agreed production level—output quotas—determined for this particular market, *given their output levels in all other markets.* Here, we assume that the standard Cournot competitive hypothesis applies: All members of a cartel operating in a certain market decide their production levels jointly in order to maximise their total joint profits subject to the production decision of every firm *outside* the cartel (if any) on this particular market and the decisions made by *all* firms in the other markets.

Throughout, we assume that monetary transfers among cartel members are not possible. In particular, we focus our analysis on symmetric equilibria, making this assumption innocuous. Indeed, all firms in our model face the same demand in each market and use the same production technology, thus, all cartel members earn equal profits in the market where the cartel operates. Hence, there is no scope for transfers.

We formalise the behaviour of cartels and individual firms as follows.

Assumption 1 *Let* $\varnothing \neq S \subset N = \{a, b, c\}$ *be a group of firms in market* M_k*. Denote by* $q^k_S = (q_{ik})_{i \in S}$ *the vector of output levels of group members in market* M_k *and by* $q^k_{-S} = (q_{jk})_{j \notin S}$ *the vector of output levels of all firms outside* S *in market* M_k*.*
Then the group S *forms a cartel in market* M_k *by maximizing the group's collective profit over the production decisions* q^k_S*, solving the optimisation problem given by*

$$\max_{q^k_S} \left(\alpha_k - \sum_{i \in S} q_{ik} - \sum_{j \notin S} q_{jk} \right) \sum_{i \in S} q_{ik} - \tfrac{1}{2} \sum_{i \in S} \left(\sum_{\ell=1}^{m} q_{i\ell} \right)^2 \tag{2}$$

Consider a market configuration $\mathbf{\Omega} = \langle \omega_1, \dots, \omega_m \rangle$*. Then for every* $k \in \{1, \dots, m\}$ *every* $S \in \omega_k$ *is assumed to determine its collective output levels by solving the objective problem stated in (2).*

The immediate consequence of Assumption 1 is that every cartel or individual firm in a market configuration for market M_k acts to maximise its collective profits over the output levels in that particular market *given* the output decisions of all non-cartel members in market M_k and of all firms in all other markets.

Example 1 To illustrate the importance of the market configuration in deriving firms' optimal decisions in context of the decision objective (2) , we consider again $m = 2$ markets M_1 and M_2 with market configuration $\mathbf{\Omega} = \langle \{ \{a\}, \{b\}, \{c\} \}, \{ \{a\}, \{b, c\} \} \rangle$. In market configuration $\mathbf{\Omega}$ firm a acts competitively in both markets and, thus, firm a chooses its quantity vector (q_{a1}, q_{a2}) to maximize its total profits as given by:

$$\begin{aligned} \max_{q_{a1}, q_{a2}} \; & p_1\, q_{a1} + p_2\, q_{a2} - C(q_a) = \\ & = (\alpha_1 - q_{a1} - q_{b1} - q_{c1})\, q_{a1} + (\alpha_2 - q_{a2} - q_{b2} - q_{c2})\, q_{a2} - \tfrac{1}{2}\, (q_{a1} + q_{a2})^2 \end{aligned} \tag{3}$$

Like firm a, firms b and c choose their respective outputs on market M_1 unilaterally. Thus each firm $i \in \{b, c\}$ chooses q_{i1} by solving the following maximization problem:

$$\max_{q_{i1}} p_1 q_{i1} - C(q_i) = (\alpha_1 - q_{a1} - q_{b1} - q_{c1}) q_{i1} - \tfrac{1}{2}\, (q_{i1} + q_{i2})^2$$

In market M_2, instead, firms b and c choose their respective outputs (q_{b2} and q_{c2}) jointly. Thus, they consider their joint profit maximization problem:

$$\begin{aligned} \max_{q_{b2}, q_{c2}} \; & p_2(q_{b2} + q_{c2}) - C(q_b) - C(q_c) = \\ & = (\alpha_2 - q_{a2} - q_{b2} - q_{c2})\, (q_{b2} + q_{c2}) - \tfrac{1}{2}\, \big((q_{b1} + q_{b2})^2 + (q_{c1} + q_{c2})^2 \big) \end{aligned}$$

The optimization problems stated above form an exhaustive and proper description of all decision processes in this particular configuration. ♦

2.2 Core Stable Market Configurations

To determine the equilibrium number, size, composition, and location of cartels, we adopt the notion of *core stability* based on the equilibrium notion of the core to the specific cartel formation model we study here. Before we present a formal definition of our core stability concept, we introduce some auxiliary notions that are used to clarify possible deviations by firms.

Definition 2 *Let $\mathbf{\Omega} = \langle \omega_1, \omega_2, \ldots, \omega_m \rangle$ be a market configuration.*
Consider $k \in \{1, \ldots, m\}$ and $S \subset N$. We say that the coalition S ***can transition*** *from structure ω_k to structure $\widehat{\omega}_k$ on market M_k if for all members $i \in S$: $i(\widehat{\omega}_k) \subseteq S$ and for all non-members $j \notin S$: $j(\widehat{\omega}_k) = j(\omega_k) \setminus S$.*
When coalition S can transition from ω_k to $\widehat{\omega}_k$ in market M_k, we denote this by $\omega_k \to_S \widehat{\omega}_k$.

A coalition S can transition from one structure to another in a certain given market if its members can abandon existing groups that they are member of and form alternative cartels with other coalition members.

A market configuration $\mathbf{\Omega}$ is now "core stable" if there is no coalition S in any market that has the ability as well as the proper incentives to transition to an alternative structure. This is formalised as follows.

Definition 3 *A market configuration $\mathbf{\Omega} = (\omega_a, \dots, \omega_m)$ is **core stable** if there does not exist a coalition (either a cartel, or a singleton) $S \subseteq N$ and an alternative market configuration $\widehat{\mathbf{\Omega}} = \langle \widehat{\omega}_1, \dots, \widehat{\omega}_m \rangle$ such that*

(i) *for every market M_k with $\widehat{\omega}_k \neq \omega_k$, coalition S can transition to $\widehat{\omega}^k$ from ω_k, i.e., $\omega_k \to_S \widehat{\omega}_k$, and*

(ii) *$\pi_i(\widehat{\mathbf{\Omega}}) \geqslant \pi_i(\mathbf{\Omega})$ for every $i \in S$ and $\pi_j(\widehat{\mathbf{\Omega}}) > \pi_j(\mathbf{\Omega})$ for at least one $j \in S$.*

Our core stability notion presumes that a deviating coalition S can form any arbitrary partition *among its own members* in any market. If, by doing so, it can make one of its members strictly better off and the other members no worse off, the original market configuration is not core stable.

3 Identification of Core Stable Cartels

To set up a benchmark for our multimarket analysis, we first present the result for core stable cartel configurations in a single market. This confirms that indeed a paradox emerges about the benefit of cartel formation under Cournot competition. Subsequently, we consider the case of two markets and identify that a unique core stable configuration emerges in which cartels withe same membership form on each market.

3.1 Core Stability in One Market

In the single market case with linear demand and quadratic cost functions, we show that there are *no* core stable market configurations. The merger paradox lies at the very foundation of this non-existence result, as firms find it profitable to transition away from a two-firm cartel.[5] The required analysis considers three fundamental market structures:

$\omega_1 = \{\{a\}, \{b\}, \{c\}\}$**:** Our model reduces to a standard Cournot model of three firms where each firm $i \in \{a, b, c\}$ maximizes its own profits taking as given the quantity chosen by the other two firms:[6]

$$\max_{q_i} \pi_i(q) = (\alpha - q_a - q_b - q_c)\, q_i - \frac{1}{2}\, q_i^2 \tag{4}$$

The reaction function of each firm i simplifies to:

$$q_i = \frac{\alpha - Q_{-i}}{3} \tag{5}$$

[5] We acknowledge that a comprehensive analysis of the single market for a finite number of firms with linear demand and quadratic costs has been conducted by Amir and Stepanova (2009). Our set up of market demand and firm cost function are a special case of those considered by Amir and Stepanova (2009) with the value of their model parameters being $b = 1$, $c = 0$, and $d = \frac{1}{2}$.

[6] We use the simplified notation $\alpha = \alpha_1$ in the case of a single market.

where we adopt the notation $Q_{-i} = \sum_{j \neq i} q_j$. The solution to the system of three equations gives us the equilibrium quantities $q_i = \frac{\alpha}{5}$. Thus, each firm's profit in this fully competitive environment equals $\pi_i = \frac{3\alpha^2}{50}$.

$\omega_2 = \{N\}$**:** Next, we consider the case when all three firms form a single monopolistic cartel and agree on their output quotas. The joint maximization problem is formally given by:

$$\max_{q_a, q_b, q_c} \sum_{i \in N} \pi_i = (\alpha - q_a - q_b - q_c)\,(q_a + q_b + q_c) - \tfrac{1}{2}\left(q_a^2 + q_b^2 + q_c^2\right)$$

The optimal quantity profile is given by the solution of the following system of first-order conditions:

$$\alpha - 3q_i - 2q_j - 2q_k \quad = \quad 0 \qquad \text{for all} \quad i, j, k \in \{1, 2, 3\} \quad \text{with} \quad i \neq j \neq k.$$

Thus, we arrive that at the optimum quantities: $q_a = q_b = q_c = \frac{\alpha}{7}$ with respective profits $\pi_i = \frac{\alpha^2}{14}$ for all cartel members $i \in N$.

$\omega_3 = \{\,\{a\}, \{b, c\}\,\}$**:** Last, we consider a market configuration with a two-firm cartel. There are three possible cartels consisting of two firms, $\{a, b\}$, $\{a, c\}$, and $\{b, c\}$. Given the symmetry of firms, however, payoffs for cartel members and the outsider firm do not depend on the identity of the firms in the cartel. We therefore present here only the case when firms b and c operate in a cartel and firm c behaves competitively.
The profit maximization problem of the cartel members is given by:

$$\max_{q_b, q_c} \pi_b + \pi_c = (\alpha - q_a - q_b - q_c)\,(q_b + q_c) - \tfrac{1}{2}\left(q_b^2 + q_c^2\right)$$

The two first-order conditions of the cartel optimization problem are:

$$\alpha - 3q_i - 2q_j - q_a = 0 \qquad \text{for all } i, j \in \{b, c\} \text{ with } i \neq j.$$

The solution to these two linear equations implies that at the optimum $q_b = q_c = \frac{\alpha - q_a}{5}$. The optimization problem of firm a which is outside of the cartel is given by (4) with the reaction function given by (5). Using these reaction functions, we arrive at the optimal quantity for the non-cooperating firm a being $q_a = \frac{3\alpha}{13}$ and for the cartel firms being $q_b = q_c = \frac{2\alpha}{13}$. Thus, the cartel members earn $\pi_b = \pi_c = \frac{10\alpha^2}{169}$, while firm a earns $\pi_a = \frac{27\alpha^2}{338}$.

We now turn to the analysis of the stability properties of each market configuration:

- Fully competitive conditions lead to payoffs that are lower than monopolistic cartel formation, i.e., $\pi_i(\omega_1) = \frac{3\alpha^2}{50} < \pi_i(\omega_2) = \frac{\alpha^2}{14}$.
- However, $\pi_a(\omega_2) < \pi_a(\omega_3) = \frac{27\alpha^2}{338}$, which shows the instability of the monopolistic cartel.

- Finally, note that $\pi_b(\omega_3) = \pi_c(\omega_3) = \frac{10\alpha^2}{169} < \pi_b(\omega_1) = \pi_c(\omega_1) = \frac{3a^2}{50}$. This shows that a cartel of two firms in a single market is unstable as well.

We conclude that in each of the three possible market configurations of a single market there is a deviating coalition: The grand coalition provides deviating possibility in the absence of cartel; any one firm will deviate from the grand coalition with the other two firms in a cartel; and any firm member of a two-firm cartel deviates to be a singleton. This implies that there indeed does not exist a core stable market configuration in a single market with three firms.

3.2 Core Stability in Two Markets

Consider the specific case where $m = 2$ with $\mathcal{M} = \{M_1, M_2\}$. Furthermore, we recall that $\alpha_1 < \alpha_2$. Even in this simplified setting, there emerge a relatively large number of non-trivial cartel formation scenarios regarding the degree of cooperation and its market location. Exploiting the homogeneity of firms, we note that agreements are distinguishable along two dimensions—the size of the cooperating group and the market on which cooperation occurs. In the case of two markets and three firms considered here, we arrive at 64 possible market configurations.

Consider a bijection $\phi\colon \{a, b, c\} \to N$. Using ϕ, we identify three fundamentally different market structure types: $\omega = \{\,\{a\}, \{b\}, \{c\}\,\}$ denotes a market structure where all firms compete in quantity, i.e., no cartel is formed; $\omega = \{\,\{\phi(a)\,\phi(b)\}, \{\phi(c)\}\,\}$ denotes a market structure where exactly two firms—denoted as $\phi(a)$ and $\phi(b)$—form a cartel and the cartel competes with the third, remaining firm (which we call a singleton) á la Cournot; and, $\omega = \{N\} = \{\,\{a, b, c\}\,\}$ denotes the case where all market participants form a single, monopolistic cartel.

Combining these possible market structures, we arrive at ten exhaustive market configurations, denoted by $\mathbf{\Omega}^n = \langle \omega_1^n, \omega_2^n \rangle$ for $n \in \{1, \ldots, 10\}$. These configurations are collected in Table 1.

$\mathbf{\Omega}^n$	ω_1^n	ω_2^n
$\mathbf{\Omega}^1$	$\{\,\{a\}, \{b\}, \{c\}\,\}$	$\{\,\{a\}, \{b\}, \{c\}\,\}$
$\mathbf{\Omega}^2$	$\{N\}$	$\{N\}$
$\mathbf{\Omega}^3$	$\{N\}$	$\{\,\{a\}, \{b\}, \{c\}\,\}$
$\mathbf{\Omega}^4$	$\{\,\{a\}, \{b\}, \{c\}\,\}$	$\{N\}$
$\mathbf{\Omega}^5$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$	$\{\,\{a\}, \{b\}, \{c\}\,\}$
$\mathbf{\Omega}^6$	$\{\,\{a\}, \{b\}, \{c\}\,\}$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$
$\mathbf{\Omega}^7$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$
$\mathbf{\Omega}^8$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$	$\{\{\phi(a), \phi(c)\}, \{\phi(b)\}\}$
$\mathbf{\Omega}^9$	$\{N\}$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$
$\mathbf{\Omega}^{10}$	$\{\{\phi(a), \phi(b)\}, \{\phi(c)\}\}$	$\{N\}$

Table 1: Possible competitive market configurations

Alternatively, we can dispense with the symbolism ϕ and show directly how these ten market configurations emerge as a result of different market structures in markets M_1 and M_2. This is represented in the following table.

ω_1^n \ ω_2^n	$\{N\}$	$\{\{a,b\},\{c\}\}$	$\{\{a,c\},\{b\}\}$	$\{\{b,c\},\{a\}\}$	$\{\{a\},\{b\},\{c\}\}$
$\{N\}$	$\mathbf{\Omega^2}$	$\mathbf{\Omega^9}$	$\mathbf{\Omega^9}$	$\mathbf{\Omega^9}$	$\mathbf{\Omega^3}$
$\{\{a,b\},\{c\}\}$	$\mathbf{\Omega^{10}}$	$\mathbf{\Omega^7}$	$\mathbf{\Omega^8}$	$\mathbf{\Omega^8}$	$\mathbf{\Omega^5}$
$\{\{a,c\},\{b\}\}$	$\mathbf{\Omega^{10}}$	$\mathbf{\Omega^8}$	$\mathbf{\Omega^7}$	$\mathbf{\Omega^8}$	$\mathbf{\Omega^5}$
$\{\{b,c\},\{a\}\}$	$\mathbf{\Omega^{10}}$	$\mathbf{\Omega^8}$	$\mathbf{\Omega^8}$	$\mathbf{\Omega^7}$	$\mathbf{\Omega^5}$
$\{\{a\},\{b\},\{c\}\}$	$\mathbf{\Omega^4}$	$\mathbf{\Omega^6}$	$\mathbf{\Omega^6}$	$\mathbf{\Omega^6}$	$\mathbf{\Omega^1}$

3.2.1 Payoff schedules

An individual firm's decision on whether to enter into a cartel agreement with other firms, where to locate the agreed cartel, and whether to leave a cartel is based solely on that firm's profits. Before developing a complete analysis, we illustrate the computations involved by considering the particular market configuration discussed in Example 1 above.

Example 1 considers a market configuration that is denoted as $\mathbf{\Omega}^6 = \langle \{\{a\},\{b\},\{c\}\}, \{\{a\},\{b,c\}\} \rangle$ in the discussion above. In the series of profit maximization problems in Example 1 that describe how firms take their decisions in the context of market configuration $\mathbf{\Omega}^6$, result into a system of six first-order conditions that are relevant for this case. More specifically, the optimization problem for firm a, which acts non-cooperatively on both markets, results in the following reaction functions:

$$\begin{aligned} q_{a1} &= \frac{\alpha_1 - q_{b1} - q_{c1} - q_{a2}}{3} \\ q_{a2} &= \frac{\alpha_2 - q_{b2} - q_{c2} - q_{a1}}{3} \end{aligned}$$

Similarly, the optimization problems that describe the decision making for firms b and c, respectively, on market M_1 yield the following reaction functions for market M_1:

$$\begin{aligned} q_{b1} &= \frac{\alpha_1 - q_{a1} - q_{c1} - q_{b2}}{3} \\ q_{c1} &= \frac{\alpha_1 - q_{a1} - q_{b1} - q_{c2}}{3} \end{aligned}$$

Last, the joint maximization problem that the cartel bc resolves on market M_2 leads to the following first-order conditions:

$$\begin{aligned} q_{b2} &= \frac{\alpha_2 - q_{a2} - 2q_{c2} - q_{b1}}{3} \\ q_{c2} &= \frac{\alpha_2 - q_{a2} - 2q_{b2} - q_{c1}}{3} \end{aligned}$$

Solving the resulting system of six equations, we arrive at the following optimal quantity profiles:

$$q_{a1} = \tfrac{1}{14}(3\alpha_1 - \alpha_2) \quad q_{b1} = q_{c1} = \tfrac{1}{49}(10\alpha_1 - \alpha_2)$$

$$q_{a2} = \tfrac{5}{98}(\alpha_2 - \alpha_1) \quad q_{b2} = q_{c2} = \tfrac{3}{98}(5\alpha_2 - \alpha_1)$$

Clearly, firms b and c curtail their outputs at the optimum in the interest of their joint profits. This results into the following total firm profits:

$$\pi_a(\mathbf{\Omega^6}) = \frac{297\alpha_1^2 - 128\alpha_1\alpha_2 + 418\alpha_2^2}{4802}$$

$$\pi_b(\mathbf{\Omega^6}) = \pi_c(\mathbf{\Omega^6}) = \frac{3(375\alpha_1^2 - 26\alpha_1\alpha_2 + 359\alpha_2^2)}{19208}$$

Similarly straightforward, though fairly tedious, computations result in the determination of all profits for cartel members and outsiders in each market configuration. These payoff schedules are collected in Table 2.[7]

In the table, the index $n \in \{1, \ldots, 10\}$, $\mathbf{\Omega}^n$ refers to the market configuration listed in Table 1. Some configurations accommodate multiple Cournot equilibria. Here, in the interest of comparability across configurations and following the adopted convention in the literature, we focus on symmetric equilibria only. This leads to a unique payoff for each player in the game for each configuration. Due to the symmetry of the firms, we only make a distinction between the profits of cartel members and outsiders. For the sake of clarity we use a subscript C to denote the profits of a cartel member and a subscript O to denote the profits of an outsider. Hence, subscript C indicates a firm that is a member of at least one cartel; subscript O identifies a firm that is not a member of cartel in any market. Furthermore, we use superscripts to denote the location of the cartel in the reported equilibrium: superscripts 1 or 2, refer to cartels on markets M_1 or M_2, respectively (but not the other) while superscript $1, 2$ indicates that firm is a member of cartels on both markets.

[7] The supporting computations of the payoff schedules presented in Table 2 are available at https://sites.google.com/site/subhadipchakrabarti/cgl_paper3 .

Case	Cartel Member	Outsider
$\mathbf{\Omega}^1$		$\pi_O = \frac{17\alpha_1^2+17\alpha_2^2-2\alpha_1\alpha_2}{288}$
$\mathbf{\Omega}^2$	$\pi_C^{1,2} = \frac{7\alpha_1^2+7\alpha_2^2-2\alpha_1\alpha_2}{96}$	
$\mathbf{\Omega}^3$	$\pi_C^1 = \frac{42\alpha_1^2+35\alpha_2^2-10\alpha_1\alpha_2}{578}$	
$\mathbf{\Omega}^4$	$\pi_C^2 = \frac{35\alpha_1^2+42\alpha_2^2-10\alpha_1\alpha_2}{578}$	
$\mathbf{\Omega}^5$	$\pi_C^1 = \frac{3(359\alpha_1^2+375\alpha_2^2-26\alpha_1\alpha_2)}{19208}$	$\pi_O = \frac{418\alpha_1^2+297\alpha_2^2-128\alpha_1\alpha_2}{4802}$
$\mathbf{\Omega}^6$	$\pi_C^2 = \frac{3(375\alpha_1^2+359\alpha_2^2-26\alpha_1\alpha_2)}{19208}$	$\pi_O = \frac{297\alpha_1^2+418\alpha_2^2-128\alpha_1\alpha_1}{4802}$
$\mathbf{\Omega}^7$	$\pi_C^{1,2} = \frac{485\alpha_1^2+485\alpha_2^2+2\alpha_1\alpha_2}{8712}$	$\pi_O = \frac{193\alpha_1^2+193\alpha_2^2-98\alpha_1\alpha_2}{2178}$
$\mathbf{\Omega}^8$	$\pi_C^1 = \frac{110\alpha_1^2+149\alpha_2^2-79\alpha_1\alpha_2}{1521}$ $\pi_C^2 = \frac{149\alpha_1^2+110\alpha_2^2-79\alpha_1\alpha_2)}{1521}$ $\pi_C^{1,2} = \frac{5(\alpha_1+\alpha_2)^2}{169}$	
$\mathbf{\Omega}^9$	$\pi_C^{1,2} = \frac{87\alpha_1^2+63\alpha_2^2+7\alpha_1\alpha_2}{1250}$ $\pi_C^1 = \frac{99\alpha_1^2+126\alpha_2^2-86\alpha_1\alpha_2}{1250}$	
$\mathbf{\Omega}^{10}$	$\pi_C^{1,2} = \frac{63\alpha_1^2+87\alpha_2^2+7\alpha_1\alpha_2}{1250}$ $\pi_C^2 = \frac{126\alpha_1^2+99\alpha_2^2-86\alpha_1\alpha_2}{1250}$	

Table 2: Equilibrium profits in each configuration

3.2.2 Core Stable configurations

In contrast to the single market case, in the two-market setting a stable cartel agreement exists provided that markets are sufficiently similar in size.[8] Interestingly, that core stable agreement is unique and hinges upon the existence of cartels with the same membership in both markets. Intuitively, our result is achieved without the need to rely on punishment strategies, entry deterrence, or intertemporal considerations discussed in the literature. Below we present formally our main result.

Proposition 1 *For $\frac{379}{523}\alpha_2 < \alpha_1 < \alpha_2$ there exists a unique core stable market configuration given by*

$$\mathbf{\Omega}^* = \mathbf{\Omega}^7 = \langle \{ \{\phi(a), \phi(b)\}, \{\phi(c)\} \}, \{ \{\phi(a), \phi(b)\}, \{\phi(c)\} \} \rangle .$$

Sketch of a proof: Assuming $\frac{379}{523}\alpha_2 < \alpha_1 < \alpha_2$ we determine exactly the incentivised transitions that coalitions can establish between the different structures in the two markets. Using tedious computations the transitions between configurations in $\{\mathbf{\Omega}^1, \ldots, \mathbf{\Omega}^{10}\}$ can be determined that are feasible—stated as condition (i) in Definition 2—as well as incentivised—stated as condition (ii) in Definition 2. These incentivised transitions are collected and

[8]It should be pointed out that, evidently, if both markets have identical sizes, i.e., $\alpha_1 = \alpha_2$, the result of the non-existence of core stable market configuration is reinstated.

represented in the transition graph in Figure 1 below. We will also adopt the short-hand notation $\pi_i(\mathbf{\Omega^n})$ to refer to profits of firm i in market configuration $n \in \{1,\dots,10\}$ which are listed in Table 2.

Hence, assuming $\frac{379}{523}\alpha_2 < \alpha_1 < \alpha_2$, in the transition graph $\mathbf{\Omega}^p - S \longrightarrow \mathbf{\Omega}^q$ for $p,q \in \{1,2,\dots,10\}$ with $p \neq q$ means that coalition S can transition in both markets: $\omega_1^p \to_S \omega_1^q$ as well as $\omega_2^p \to_S \omega_2^q$ and, furthermore, coalition S benefits from this transition in the sense that $\pi_i\left(\mathbf{\Omega}^q\right) \geqslant \pi_i\left(\mathbf{\Omega}^p\right)$ for every $i \in S$ and $\pi_j\left(\mathbf{\Omega}^q\right) > \pi_j\left(\mathbf{\Omega}^p\right)$ for at least one $j \in S$.

All other transitions are not depicted in the graph in Figure 1. Hence, all transitions that are *not* represented in Figure 1 are either not feasible, or are feasible, but not incentivised.

From the incentivised transitions depicted in Figure 1, it is clear that configuration $\mathbf{\Omega}^7$ represents a sink in this graph. This means that $\mathbf{\Omega}^7$ is indeed a core stable configuration.

Moreover, from the observation that any alternative configuration $\mathbf{\Omega}^p$, $p \neq 7$, is connected to $\mathbf{\Omega}^7$ through a directed path of incentivised transitions in the graph and that there are no cycles or loops in this graph, it follows that $\mathbf{\Omega}^7$ is actually the unique core stable configuration.

This completes the sketch of the proof of the assertion of the proposition. ■

`Insert Figure 1 here.`

3.3 Discussion: One Market versus Two Markets

We would like to provide some intuition for our results, and, in particular we will focus our discussion on the role of convexity. The reason we do not get a core stable market configuration in the one market case is because of two mechanisms. Merger paradox refers to the situation where partial cooperation makes the cooperators worse off compared to no cooperation (i.e., fully competitive environment). The free rider problem refers to the situation in which cooperators under full cooperation are worse off compared to non-cooperators under partial cooperation. The first results in a deviating singleton for a two-firm cartel. The second results in a deviating singleton for the grand coalition.

With two markets, these effects do not disappear but are *weakened.* The merger paradox and the free rider problem both hold unambiguously only in market configuration $\mathbf{\Omega}^9$: In Figure 1 the merger paradox is captured by the transition from $\mathbf{\Omega}^9$ to $\mathbf{\Omega}^3$ while the free rider problem drives the transition from $\mathbf{\Omega}^9$ to $\mathbf{\Omega}^7$. Similarly, we can attribute to either the merger paradox or free rider problems all transitions depicted in Figure 1.

Recall that we stated in the introduction that convexity of the cost function has received some attention in the literature. The reason is that this property of the production technology may be employed to weaken the merger paradox. Consider a general quadratic cost function given by

$$C(q_i) = \gamma \left(\sum_{k=1}^{m} q_{ik} \right)^2 \quad \text{for some } \gamma > 0.$$

Note that for our analysis thus far we have used $\gamma = \frac{1}{2}$. Consider the range of values of γ for which a cartel member in a two firm cartel successfully deviates and forms a singleton (merger paradox) in a single market interaction (i.e., for $m = 1$). Contrast that with the range of values of γ for which the same situation occurs in a two-market setting (i.e., $m = 2$). From our analysis so far, we know that if $\frac{379}{523}\alpha_2 < \alpha_1 < \alpha_2$, $\gamma = \frac{1}{2}$ belongs to the former range but not the latter: as there is no core stable cartel in the single market case whereas $\mathbf{\Omega}^7$ is a core stable market configuration for the cartel for the two-market case. In fact, one can show that the latter range is a strict subset of the former range.

Assume that $\alpha_1 = 100$ and $\alpha_2 = 125$. These parameter values satisfy the inequality in our proposition. One can show that in the one market case ($m = 1$), there is *no* merger paradox type for $0 < \gamma < 0.801938$.

Next, consider the market configuration $\mathbf{\Omega}^7$ in the two market case ($m = 2$). There are three types of deviations possible. A member of the cartel can deviate in market M_1 forming a singleton and resulting in market configuration $\mathbf{\Omega}^6$. It can deviate in market M_2 resulting in market configuration $\mathbf{\Omega}^5$. It can deviate in both markets resulting in market configuration $\mathbf{\Omega}^1$. The range of γ for successful deviations in the first case is $0 < \gamma < 0.210087$, in the second case is $0 < \gamma < 0.399714$, and in the third case is $0 < \gamma < 0.430677$. The reduced range of γ in all three cases testifies to this weakening.

4 Further Remarks

The contribution of our work is to demonstrate that stable cooperation agreements may exist when firms interact in more than one market whereas under the same conditions in a single-market interaction there are no stable agreements. Notably, we identify a unique pattern of stable agreements: they are partial as opposed to involving all market participants; and they operate in both markets and among the same firms. The main mechanism underpinning our results is the weakening of the merger paradox, i.e., the incentives of a firm to break apart in a two-firm cartel.

Two important assumptions deserve some more attention. Firstly, as pointed out in Section 2, the cooperative agreements studied here are localized in a specific market. That is in our model a cartel is operating on a single market with cartel members agreeing on the quantity quotas to that specific market. On the one hand, this assumption justifies our choice to focus on symmetric equilibria given that firms face the same technology and the same market conditions. Alternatively, a "global cartel" could be modelled as an agreement that commits firms to operate on some markets and not on others. Schröder (2007) analyse the stability of international cartels and demonstrate that the viability of such agreements rely on repeated interaction, the ability to punish non-compliant members of the cartel, and the size and form of transaction costs that link markets.

On the other hand, the focus on local agreements weakens concerns about enforcement and monitoring. The argument is linked to the second important assumption that we are making, that is that cartel members do not share their profits, i.e., payoffs are non-transferable. This assumption too fits well with our approach to focus on symmetric equilibria as by definition cartel members will be earning the same profits on the market where the cartel operates, hence, no firm member of a cartel earns higher payoffs than other members which the higher-profit firm can use to compensate any lower-profit member of the cartel and ensure their cooperation. We would argue that cooperative agreements that rely on self-administering of side payments are more prone to deviations—a firm may renege on giving out the side payment or a recipient of side payments may renege on the agreed strategy—as such agreements would need to rely on long-term repeated interactions.

Both assumptions—the local nature of the cartel and the non-transferability of payoffs—allow us to highlight that cooperative arrangements among firms in multimarket contact environments can be self-sustained under more general conditions and weak institutional settings than previously studied in the literature.

Opportunities for future work are manifold. As a first step one may generalise this model to an arbitrary number of firms and markets. The challenge of such a model—as discussed earlier—is in the increased complexity of possible cooperative agreements. In this more general context, concepts developed in network theory may be useful to render these more general models tractable.

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