Globalisation and Inequality:
A Yangian General Equilibrium Analysis*

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Abstract

Inequality has become a much debated issue in the ongoing assessment of the performance of the contemporary post-crisis global economy. In this paper I present a Yangian general equilibrium approach that shows that global trade institutions might result in significantly higher welfare inequality than an organic matching economy based on an endogenous social division of labour. Global markets enhance the effects of social scarcity of the most preferred commodity, thus increasing the welfare of its primary producers over the agents producing the less socially scarce commodity. A detailed analysis is presented to identify the institutional features that cause this opportunity inequality.

1 Introduction

Recently, public discussion in the media and the popularity of publications such as Stiglitz (2013) and Piketty (2014) have drawn our attention to the issue of welfare inequality in our contemporary global economy. Inequality has long been recognised as a valid subject of research in economics and, in particular, in general equilibrium theory. In the 1970s a large theoretical literature on inequality and fairness developed. These contributions focussed on developing concepts that expressed fairness as the absence of envy, in particular initiated from the seminal concept of *envy-freeness* (Foley, 1967). This theory strictly avoids the use of interpersonal utility comparisons as well as the measurement of income inequality, so much the subject of the recent public debate on inequality. Based on this literature a theory of fairness in market economies was developed through seminal contributions by Kolm (1972); Schmeidler and Vind (1972); Varian (1974); Pazner (1977); Pazner and Schmeidler (1978) and Thomson (1983).[1]

The traditional general equilibrium approach to fairness has some deficiencies that makes it less effective as a tool to consider the issues raised recently about inequality in our contemporary global economy. First, the fairness concepts introduced have a rather limited scope, since they are based on a setting in which agents are only compared through subjective envy of other agents' allocations.

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[1] The main result from this theory is that every competitive (Walrasian) equilibrium from equal endowments is envy-free and efficient.

So, particularly, the scope of this literature avoids addressing inequality through income comparisons. In this paper we impose a very restrictive assumption by endowing all agents with exactly the same utility function, thereby making it possible to do comprehensive and exhaustive welfare comparisons. Although restrictive, this hypothesis is unavoidable to make meaningful inferences about inequality in a market setting.

Second, the application of a Walrasian general equilibrium framework imposes ex-ante the strict dichotomy between consumption and production, thereby excluding the consideration of how the social division of labour might be affected through institutional change. In this paper, I explicitly use a Yangian (Yang, 1988, 2001) framework in which all agents are modelled as consumer-producers, implying that there emerges an endogenous social division of labour in the economy. This allows us to implement a rich environment through relatively simple means, in which a wide variety of qualitative inferences can be made from the equilibria that emerge in the economy. In particular, I focus on how the endogenous adjustment of the social division of labour affects welfare inequality under different institutional trade infrastructures.

I focus in this paper on an alternative approach to measure inequality in a general equilibrium framework that addresses the conversion from localised trade to globalised trade. My starting point is the well-accepted proposition that more global trade leads generally to higher welfare and increased efficiency. This assertion is based on the direct extension of insights from Walrasian general equilibrium theory. Indeed, the First Welfare Theorem (Debreu, 1951) states that in a Walrasian system of perfectly competitive markets, trade results in an efficient allocation in the sense of Pareto (1906). This was refined later to more differentiated forms of efficiency. It is natural to conclude, therefore, that if markets encompass more traders, the resulting equilibria lead to higher overall welfare due to increased efficiency. The Yangian perspective—founded on the emergence of an endogenous social division of labour—does not alter this insight.

However, the issue of how the generated wealth is distributed, is not subject to the normative notion of Pareto efficiency. The generated wealth can be distributed unevenly, even though there is an increased efficiency in the economy. Welfare inequality, therefore, remains a valid point of concern under globalisation. This is well-accepted in international trade theory based on the Heckscher-Ohlin model (Heckscher, 1919; Ohlin, 1933) that inequality can increase as a consequence of the engagement in trade.

In this paper I extend this perspective into the Yangian theory of general equilibrium that centres on the endogenous emergence of a social division of labour. We can apply this framework to various institutional trade environments and compare the resulting wealth distributions. Through this approach we can address the influence of institutional trade infrastructures on inequality. We look particularly at two very different institutional frameworks within which a global social division of labour emerges.

First, we look at a matching network in which the scope of trade is minimal—denoted as a *matching economy*. This institutional trade environment is modelled as a three-stage process. In the first stage economic agents select trading partners; in the second stage all matched partners specialise through the selection of a certain production plan; and, finally, in the third stage trade is engaged between the matched individuals. A social division of labour emerges in which economic agents specialise optimally and engage only in trade with opposite specialists. Under certain conditions, there can emerge an economy with matchings between autarkic farmers, representing an autarkic rival sector in the economy.

Second, we consider a standard Yangian general equilibrium framework (Yang, 2001), representing a global market economy. A global price now guides all economic agents simultaneously to specialise in certain production plans, determined by their individual productive abilities. In the emerging global equilibria, all economic agents always engage in trade and, thus, there does not emerge an autarkic, rural sector in the economy.

Within this framework, we aim to investigate welfare inequalities across these two institutional trade infrastructures by considering the general equilibria that arise under both institutional settings. In both institutional trade infrastructures, significant welfare inequalities emerge among the different types of economic agents: Producers of necessary goods (farmers) *always* attain higher wealth than the (specialised) producers of luxury goods.

There are, however, significant differences in the wealth distributions under these two institutional trade infrastructures. In general, if there are higher returns to specialisation, the identified welfare inequality between these types is more pronounced in a global Yangian equilibrium than in an equilibrium in the matching framework. This implies that generally globalisation enhances welfare inequalities, if there are higher returns from specialisation. Only if output levels under specialisation are limited, welfare inequalities might abate due to globalisation.

A secondary, less pronounced factor in the determination of whether globalisation results in increased inequality is preferential: The more the necessary good is appreciated by the agents in the economy, the more the inequality increase is abated. For high enough preference of the necessary good, the resulting equilibrium in a Yangian global market economy is fully autarkic, meaning that all consumer-producers specialise as farmers and there is no trade.

The Yangian general equilibrium approach. In this paper our approach centres on the endogenous emergence of a social division of labour in the sense of Yang (1988); Yang and Borland (1991) and Yang (2001, 2003). This approach founds wealth creation explicitly on the combination of increasing returns to specialisation with the principle of gains from trade in an economy consisting of consumer-producers.

Here the concept of *increasing returns to specialisation* refers to the human ability to achieve higher output levels if human capital is concentrated on skills related to a limited set of productive tasks. This implies that human capital is more developed in the productive skills related to a limited set of goods. Mathematically this is represented by non-convexities in the production set, in particular along the axes of the commodity space.[2] Trade among specialised economic agents now exploits the identified increasing returns to specialisation and achieves the generation of significant social wealth.

The Yangian approach uses the powerful notion of a *consumer-producer*—which combines consumptive and productive abilities into a single representative concept—rather than the classical di-

[2]It should be pointed out that increasing returns to specialisation is a generally weaker concept than increasing returns to scale. For an exposition of this I refer to Diamantaras and Gilles (2004).

chotomy of production and consumption through separated sets of consumers and producers. Each consumer-producer is endowed with a utility function as well as a production set, which is subject to increasing returns to specialisation. Under a price mechanism with price-taking behaviour, these consumer-producers will specialise in a corner-solution of the production optimisation problem, thereby specialising in the production of a limited set of goods. (Yao, 2002; Cheng and Yang, 2004; Diamantaras and Gilles, 2004)

In this paper, our simple model endows all agents with exactly the *same* utility function, thus facilitating a complete analysis of welfare inequalities generated in the two different institutional trade environments. Agents are differentiated according to their productive abilities, introducing two different types of agents—one type predisposed to produce a highly desirable, necessary good (Type A) and the other type more adept to produce a luxury good (Type B).

In this setting we compare a matching economy in which trade relationships are completely localised between two agents only and conducted at competitive prices. The other institutional setting is that of a system of perfectly competitive global markets. In both systems Type A agents have higher welfare than Type B agents. However, inequalities increase under a global market system for certain parameter values. In particular, one can conclude that global markets put more weight on the socially scarcer commodity through a higher price as well as through a modified social division of labour that reduces the total supply of that good. This is a consequence of the social division of labour to be guided by the price mechanism rather than relationally through a network of trade relationships.

Structure of the paper: In the next section an overview of the Yangian foundations of the model are presented, introducing the details of the model of a consumer-producer. Section 3 debates a matching economy based on bilateral interaction. Section 4 looks at the Yangian general equilibrium model of a global economy and Section 5 carefully analyses welfare inequalities in the two introduced systems. We conclude with the discussion of the resulting insights.

2 The fundamentals

All agents are consumer-producers. In principle we assume that there is a continuum of agents with mass 1 (unity) and that we can express different types of agents in fractions of the total population.

There are two goods: X stands for food stuffs and Y stands for cloth—or any generic luxury commodity.

Consumption: All agents have an identical utility function given by a Stone-Geary formulation with

$$U(x, y) = x(y + \alpha) \tag{1}$$

where $\alpha > 0$ describers the luxury nature of the Y-good and hoe much foodstuffs X are preferred to the luxury commodity Y.

It is clear that the selection of this utility function is limiting, but it has rather appealing properties based on the following considerations. Indeed, this utility function imposes that X is a necessity, while Y is a luxury good. This allows for a relatively rich structure, since it supports the emergence of an equilibrium in which all economic agents specialise as farmers, but it does not support an equilibrium in which all consumer-producers are weavers. On the other hand, in a CES formulation both goods would essentially be a luxury good, while in a Cobb-Douglas formulation both goods become necessities.

Production: There are two types of agents in this economy, that are differentiated from each other through their productive abilities. These two types represent differentiated human capital in the population. Simply, the two types of agents are specialised—or "educated"—in different specialisations, namely as farmers or as weavers. Of course, agents who are trained as farmers can produce cloth (Y) as well, but at a lower output level than weavers. Similarly, weavers are able to farm, but typically at a lower output level than farmers.[3]

Assuming that $Q \geqslant 1$ represents full productive ability in any of the two goods, the productive outputs are now formulated as follows:

Agent type	Specialisation X	Specialisation Y	Population fraction
Type A	$(Q, 0)$	$(0, 1)$	$0 < f < 1$
Type B	$(1, 0)$	$(0, Q)$	$0 < 1 - f < 1$

The determination of the fraction f is institutional in this framework. Indeed, one can think of an institutional barrier to freely train oneself as a Type A farmer. In particular in our analysis we are interested in the case that the class of farmers is a minority, i.e., $0 < f \leqslant \frac{1}{2}$, implying that the fraction of weavers—representing luxury good producers—is a majority. This restriction generates a rather interesting framework for the emergence of increased inequality due to globalisation as we see in Section 4.

The restriction that $f \leqslant \frac{1}{2}$ can be founded on a certain view of the ability to become a farmer in the primitive economy considered here. It is founded on the hypothesis that there is a fundamental, fixed quantity of a certain input for the production of X, say land. In the feudal system it was imposed that only the eldest surviving son of a landowner could inherit the family estate, while the younger male siblings were employed elsewhere.[4] The limited quantity of arable land in combination with the inheritance institution described results in a strict upper bound on the fraction of landowning farmers.[5]

[3] The education of these agents is subject to a potential extension of this model. Indeed, one can ask how these agents are typically educated. An overlapping generations framework can be constructed to model the process of education and the training of a new generation of agents by an existing generation of economic agents. Two educational systems can be implemented here, namely an apprentice system placing a novice with a master specialist and a public education system, where education is performed through a professional class of educators.

[4] In Anglo-Saxon noble landholding families, younger sons were usually directed to a career in the Roman Catholic church. Female offspring was usually used for purposes of building a social network for the family.

[5] The majority fraction $(1 - f)$ of weavers stands now for the urban population in the society that can either produce luxury goods in the urban economy, founded on a well-developed social division of labour, and working as a peasant for

The model therefore has **three main parameters**: α; Q; and f.

3 The network economy

We consider a three-stage formulation of the production-trade process in a matching economy, representing local trade only. This three-stage process can be described as follows:

Stage 1: All agents select *one and only one* trade partner and form a matching network of trade relationships—the trade *infrastructure.*

Stage 2: All agents select a production plan, given their productive abilities corresponding to their type, as reported in the table above. The selected production plan is executed.

Stage 3: All paired agents engage in trade under a competitive price mechanism. Hence, given the produced quantities of the two commodities in the trade relationship, both agents barter and establish an exchange rate that equates demand and supply in that relationship. This competitive price reflects the social scarcity of the two goods in that trade relationship properly. Final utility levels are established after consumption after trade.

The use of a competitive price emerging in Stage 3 of this trade process is founded on the underlying hypothesis that the two traders are equally powerful and that the barter process is balanced. So, none of these traders is able to impose conditions on the other trader. In this case it seems plausible and justifiable to implement simply a competitive price as the outcome of the trade process in Stage 3.[6]

Next, backward induction is used to compute the subgame perfect Nash equilibrium (SPNE) for this particular three-stage model. First, we can determine for any configuration of output levels the resulting final consumption bundles in a bilateral trade relationship under price-taking behaviour for the given utility function.

Next, we can determine for any potential bilateral matching the optimal specialisations, given the final consumption bundles resulting from trade. Every potential bilateral trade relationship can be represented in a normal form matrix game in which agents' strategies are given as the available production plans. Nash equilibria can be identified in all matrix form representations of such typified trade relationships. Finally, an equilibrium matching network pattern can be identified from the equilibria determined in the second stage of the three-stage matching framework.

Setup and notation: Throughout we use food stuffs X as money and express the value of the luxury good Y in terms of X. Hence, $p \geqslant 0$ describes the quantity of X that is equivalent to one unit of Y. Or, p is the price of Y in terms of X.

a landholding farmer.

[6] An alternative would be to implement a non-price trading process such as Edgpeworthian barter represented by a core allocation. Noting that any competitive equilibrium allocation is in the core, also this interpretation would allow for the use of a competitive price in Stage 3 of this matching equilibration process.

3.1 Solving Stage 3

From the consumption problem for a given level of income $I > 0$ we derive from the interior solutions based on first order conditions that demand functions for both goods are given by

$$d_X(p) = \frac{I + \alpha p}{2} \tag{2}$$

$$d_Y(p) = \frac{I - \alpha p}{2p} \tag{3}$$

These demand functions will be used throughout to derive equilibrium prices and utility levels for all binary trade situations.

Indeed, if two traders meet that have supply vectors $(X, 0)$ and $(0, Y)$, then in their trade relationship clearly we have that $I_X = X$ and $I_Y = pY$. Hence,

$$d^X = \left(\frac{X + \alpha p}{2}, \frac{X - \alpha p}{2p} \right) \tag{4}$$

$$d^Y = \left(\frac{p(Y + \alpha)}{2}, \frac{Y - \alpha}{2} \right) \tag{5}$$

These results imply that we have to impose two constraints to avoid boundary solutions to the consumption problem: $0 \leqslant \alpha \leqslant Y$ and $0 \leqslant \alpha \leqslant \frac{X}{p}$. The first condition is binding, but the second is immaterial as shown below and can be disregarded in the following analysis.

This leads to the conclusion that for any $0 \leqslant \alpha \leqslant Y$ the trade relationship equilibrium exchange rate or price is given by

$$\hat{p} = \frac{X}{Y + 2\alpha}. \tag{6}$$

This implies that the second condition ($0 \leqslant \alpha \leqslant \frac{X}{\hat{p}} = Y + 2\alpha$) is non-binding under the equilibrium price. Post-trade final consumption bundles are now given by

$$C^X = \left(\frac{X(Y + 3\alpha)}{2(Y + 2\alpha)}, \frac{Y + \alpha}{2} \right) \tag{7}$$

$$C^Y = \left(\frac{X(Y + \alpha)}{2(Y + 2\alpha)}, \frac{Y - \alpha}{2} \right) \tag{8}$$

and equilibrium utility levels are computed as

$$U^X = \frac{X(Y + 3\alpha)^2}{4(Y + 2\alpha)} \tag{9}$$

$$U^Y = \frac{X(Y + \alpha)^2}{4(Y + 2\alpha)} \tag{10}$$

Hence, $U^X > U^Y$ if $\alpha > 0$ regardless of the values of X and Y.

The analysis above is only valid for $0 \leqslant \alpha \leqslant Y$, which is a binding constraint leading to two separate cases in Stage 2. In the case that $\alpha > Y$ we derive that

$$d^X = \left(\frac{X + \alpha p}{2}, \frac{X - \alpha p}{2p}\right) \tag{11}$$

$$d^Y = (pY, 0) \tag{12}$$

This implies that the equilibrium price is now given by

$$\hat{p} = \frac{X}{\alpha + 2Y} \tag{13}$$

leading to consumption bundles given by

$$C^X = \left(\frac{X(Y + \alpha)}{2Y + \alpha}, Y\right) \tag{14}$$

$$C^Y = \left(\frac{XY}{2Y + \alpha}, 0\right) \tag{15}$$

and equilibrium utility levels are computed as

$$U^X = \frac{X(Y + \alpha)^2}{2Y + \alpha} \tag{16}$$

$$U^Y = \frac{\alpha XY}{2Y + \alpha} \tag{17}$$

We conclude that once again, $U^X > U^Y$ for all relevant parameter values with $\alpha > Y$.

3.2 Solving Stage 2

We now investigate all potential trade relationships with regard to the selection of a production plan for each of the two traders. The three patterns we investigate are the trade relationships between two Type A agents, two Type B agents and between one Type A and one Type B agent.

We use the results from the previous analysis regarding behaviour in Stage 3 to describe outcomes in the analysis of Stage 2. Hence, appropriate values of X and Y are selected in the utility formulations in (9), (10), (16) and (17) above to formulate matrix representations of the three potential trade relationships.

Type A meets Type A: Given the production ability of 1 unit of Y for a Type A agent ("farmer") we have to impose restrictions on the parameter α in our analysis.

For $0 \leqslant \alpha \leqslant 1$ we derive after-trade equilibrium utility levels for different specialisation patterns represented in matrix form as

A/A	X	Y
X	$\alpha Q\,,\,\alpha Q$	$\frac{Q(1+3\alpha)^2}{4(1+2\alpha)}\,,\,\frac{Q(1+\alpha)^2}{4(1+2\alpha)}$
Y	$\frac{Q(1+\alpha)^2}{4(1+2\alpha)}\,,\,\frac{Q(1+3\alpha)^2}{4(1+2\alpha)}$	$0\,,\,0$

We derive the following matrix form representation for $\alpha > 1$:

A/A	X	Y
X	$\alpha Q\,,\,\alpha Q$	$\frac{Q(1+\alpha)^2}{\alpha+2}\,,\,\frac{\alpha Q}{\alpha+2}$
Y	$\frac{\alpha Q}{\alpha+2}\,,\,\frac{Q(1+\alpha)^2}{\alpha+2}$	$0\,,\,0$

The analysis of these two matrix representations leads us to the following conclusions concerning the resulting equilibria:

- For $0 \leqslant \alpha < \frac{1}{1+2\sqrt{2}}$ the unique Nash equilibrium is (X, Y);
- For $\alpha = \frac{1}{1+2\sqrt{2}}$ there are two Nash equilibria, (X, Y) and (X,X);
- For $\frac{1}{1+2\sqrt{2}} < \alpha \leqslant 1$ the unique Nash equilibrium is (X,X); and
- For $\alpha > 1$ the unique Nash equilibrium is (X,X) as well.

The resulting payoffs in this interaction are now deduced as

$$U = \left(\frac{Q(1+\alpha)^2}{4(1+2\alpha)}\,,\, \frac{Q(1+3\alpha)^2}{4(1+2\alpha)} \right) \qquad \text{for } 0 \leqslant \alpha \leqslant \tfrac{1}{1+2\sqrt{2}} \tag{18}$$

$$U = (\,\alpha Q\,,\,\alpha Q\,) \qquad \text{for } \alpha > \tfrac{1}{1+2\sqrt{2}} \tag{19}$$

Type B meets Type B: Given the production ability of Q unit of Y for a Type B agent ("weaver") we have to impose restrictions on the parameter α in our analysis.

For $0 \leqslant \alpha \leqslant Q$ we derive after-trade equilibrium utility levels for different specialisation patterns represented in matrix form as

B/B	X	Y
X	$\alpha\,,\,\alpha$	$\frac{(Q+3\alpha)^2}{4(Q+2\alpha)}\,,\,\frac{(Q+\alpha)^2}{4(Q+2\alpha)}$
Y	$\frac{(Q+\alpha)^2}{4(Q+2\alpha)}\,,\,\frac{(Q+3\alpha)^2}{4(Q+2\alpha)}$	$0\,,\,0$

For $\alpha > Q$ we derive the following matrix representation:

B/B	X	Y
X	$\alpha\,,\,\alpha$	$\frac{(Q+\alpha)^2}{Q+2\alpha}\,,\,\frac{\alpha Q}{Q+2\alpha}$
Y	$\frac{\alpha Q}{Q+2\alpha}\,,\,\frac{(Q+\alpha)^2}{Q+2\alpha}$	$0\,,\,0$

The analysis of these two matrix representations leads us to the following conclusions concerning the resulting equilibria:

- For $0 \leqslant \alpha < \frac{Q}{1+2\sqrt{2}}$ the unique Nash equilibrium is (X, Y);
- For $\alpha = \frac{Q}{1+2\sqrt{2}}$ there emerge two Nash equilibria, (X, Y) and (X, X);
- For $\frac{Q}{1+2\sqrt{2}} < \alpha \leqslant Q$ the unique Nash equilibrium is (X, X); and
- For $\alpha > Q$ the unique Nash equilibrium is (X, X) as well.

The resulting payoffs in this interaction are now deduced as

$$U = \left(\frac{(Q+3\alpha)^2}{4(Q+2\alpha)}, \frac{(Q+\alpha)^2}{4(Q+2\alpha)} \right) \qquad \text{for } 0 \leqslant \alpha \leqslant \frac{Q}{1+2\sqrt{2}} \tag{20}$$

$$U = (\,\alpha\,,\,\alpha\,) \qquad \text{for } \alpha > \frac{Q}{1+2\sqrt{2}} \tag{21}$$

Type A meets Type B: The most complicated case is that of a trade interaction between a Type A agent and a Type B agent. We show that the parameter space is divided into several subspaces corresponding to conditions under which different equilibrium outcomes result. A full analysis is presented below.

Note that the analysis of Stage 3 is implemented for $Y = 1$ as well as $Y = Q$ for different cases. Hence, there are two thresholds for the value of α to be considered, namely 1 and Q. Assuming that $Q \geqslant 1$ we get the regions $0 \leqslant \alpha \leqslant 1$, $1 \leqslant \alpha \leqslant Q$ and $\alpha \geqslant Q$ as the relevant parameter subspaces in our analysis.

First, for $0 \leqslant \alpha \leqslant 1$ we derive after-trade equilibrium utility levels for different specialisation patterns represented in matrix form as

A/B	X	Y
X	$\alpha Q\,,\,\alpha$	$\frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}\,,\,\frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$
Y	$\frac{(1+\alpha)^2}{4(1+2\alpha)}\,,\,\frac{(1+3\alpha)^2}{4(1+2\alpha)}$	$0\,,\,0$

Next, for $1 < \alpha \leqslant Q$ we derive the following matrix form:

A/B	X	Y
X	$\alpha Q\,,\,\alpha$	$\frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}\,,\,\frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$
Y	$\frac{\alpha}{2+\alpha}\,,\,\frac{(1+\alpha)^2}{2+\alpha}$	$0\,,\,0$

Finally, for $\alpha > Q$ we derive the following matrix representation:

A/B	X	Y
X	$\alpha Q\,,\,\alpha$	$\frac{Q(Q+\alpha)^2}{2Q+\alpha}\,,\,\frac{\alpha Q^2}{2Q+\alpha}$
Y	$\frac{\alpha}{2+\alpha}\,,\,\frac{(1+\alpha)^2}{2+\alpha}$	$0\,,\,0$

Using these three matrix representations for the trade relationship between a Type A and a Type B agent we can now derive the resulting equilibria for different parameter values of α and Q. Throughout I restrict the analysis to the range $1 \leqslant Q \leqslant 3$, which restricts the effects of education and mentoring to a threefold multiplication of the productive ability of a specialised agent in the chosen specialisation. It is particularly for these parameter values that our analysis yields interesting insights.

The analysis of the Type A – Type B trade relationship results in the identification of 6 subspaces in the parameter space for the given Q-range. This is depicted in Figure 1, which plots the subspaces in the given parameter ranges. In each of these subspaces one can list the identified properties and the equilibria resulting:

Subspace $\mathcal{A}$: This subspace is characterised by all parameter values of $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$ such that $\alpha \geqslant Q$. Thus, the interaction is described by the third matrix representation. In this parameter space one can easily check that the following two inequalities hold:

$$\frac{\alpha Q^2}{2Q+\alpha} < \alpha \tag{22}$$

$$\frac{\alpha}{\alpha+2} < \alpha Q \tag{23}$$

Hence, in this area (X,X) is the unique Nash equilibrium specialisation pattern that emerges in the trade relationship between a Type A and a Type B agent.

Subspace $\mathcal{B}'$: This subspace is characterised by all parameter values of $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$ such that $1 \leqslant \alpha < Q$ and such that additionally the following inequality holds

$$\frac{Q(Q+\alpha)^2}{4(Q+2\alpha)} < \alpha \tag{24}$$

In this area the second matrix representation of the trade relationship holds. For this area it can be easily verified that also $\frac{\alpha}{\alpha+2} < \alpha Q$, which implies that (X, X) is again the unique Nash equilibrium specialisation pattern that emerges in the trade relationship between a Type A and a Type B agent.

Subspace $\mathcal{C}'$: This subspace is characterised by all parameter values of $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$ such that $1 \leqslant \alpha < Q$ and such that additionally the following inequality holds

$$\frac{Q(Q+\alpha)^2}{4(Q+2\alpha)} > \alpha \tag{25}$$

In this area the second matrix representation of the trade relationship holds. For this area it can be easily verified that also $\frac{\alpha}{\alpha+2} < \alpha Q$, which implies that (X, Y) is here the unique Nash equilibrium specialisation pattern that emerges in the trade relationship between a Type A and a Type B agent.

Subspace $\mathcal{B}$: This subspace is characterised by all parameter values of $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$ such that $\alpha < 1$ and such that additionally the inequality (24) holds. This implies that the trade relationship is fully described by the first matrix representation introduced above. Furthermore, it can be verified that it holds that

$$\frac{(1+\alpha)^2}{4(1+2\alpha)} < \alpha Q \tag{26}$$

This implies that in this parameter subspace there emerges a unique Nash equilibrium in the trade relationship, being the autarkic farming specialisation pattern (X, X).

Subspace $\mathcal{C}$: This subspace is characterised by all parameter values of $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$ such that $\alpha < 1$ and such that additionally the inequalities (25) as well as (26) hold. This implies that the trade relationship is fully described by the first matrix representation introduced above.
This implies that in this parameter subspace there emerges a unique Nash equilibrium in the trade relationship, being the mutual specialisation pattern (X, Y).

Subspace $\mathcal{D}$: Finally, this subspace is characterised by all parameter values of $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$ such that $\alpha < 1$ and such that additionally

$$\frac{(1+\alpha)^2}{4(1+2\alpha)} > \alpha Q \tag{27}$$

Again, this implies that the trade relationship is fully described by the first matrix representation introduced above. Here it can also be verified that inequality (25) holds. Together with the above this implies that in this parameter subspace there emerge two Nash equilibrium specialisation patterns, namely (X, Y) as well as (Y, X).

In Figure 1, the parameter boundaries for the space itself ($Q = 1$ and $Q = 3$) and the various matrix representations ($\alpha = 1$ and $\alpha = Q$) are depicted as red lines. The coloured areas result from inequalities (24) and (26).

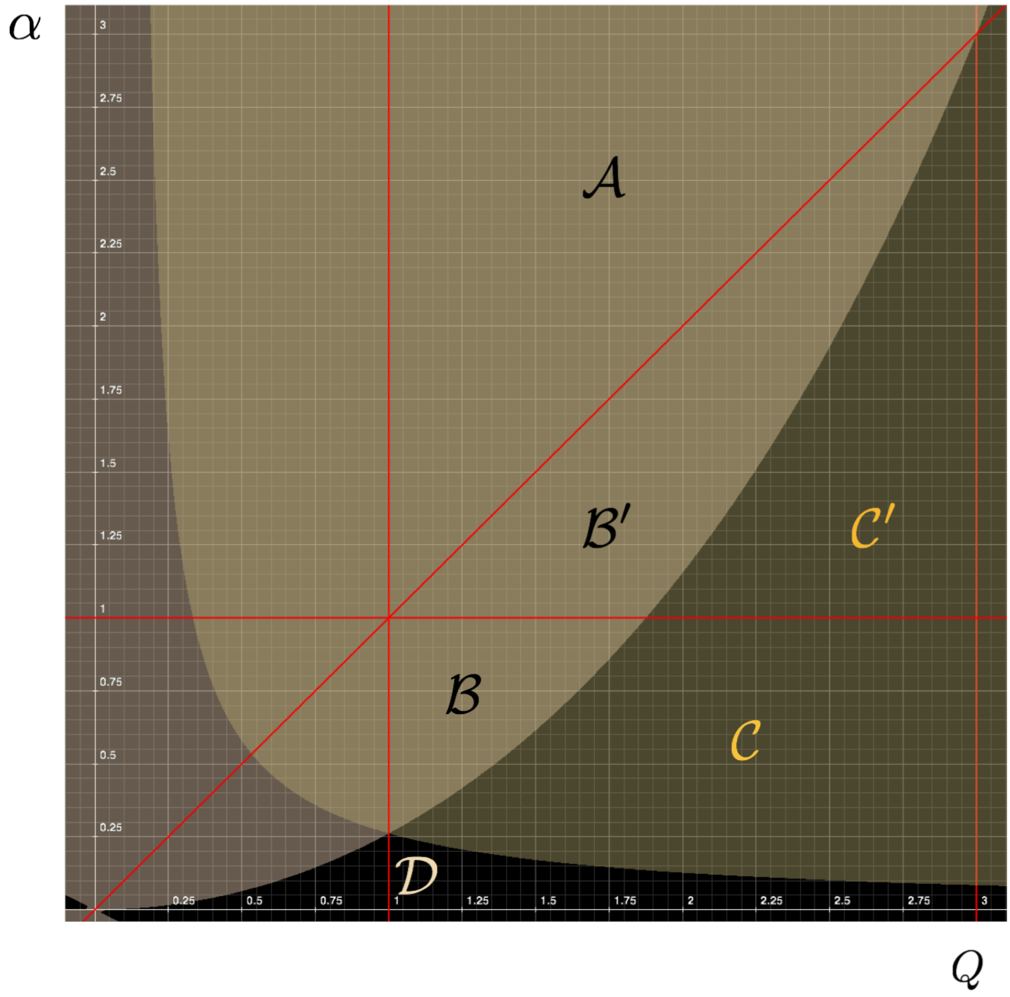

Figure 1: Parameter space for Type A – Type B interaction

The identified Nash equilibrium specialisation patterns now imply that we can reduce our findings to two conclusions. In the given parameter space described by $\alpha \geqslant 0$ and $1 \leqslant Q \leqslant 3$, we identify two relevant subspaces:

- In the subspace $\mathbb{A} = \mathcal{A} \cup \mathcal{B} \cup \mathcal{B}'$ there emerges the autarkic farming specialisation pattern (X, X) as the unique Nash equilibrium.
- In the subspace $\mathbb{B} = \mathcal{C} \cup \mathcal{C}' \cup \mathcal{D}$ there emerges the mutual specialisation pattern (X, Y) as a Nash equilibrium pattern. In the following analysis of Stage 1 of the game we focus on this equilibrium in particular.

We emphasise that although the equilibrium patterns are simple, the payoff formulations are different depending on what part of the parameter space one focusses on.

3.3 Solving Stage 1

Given the equilibria identified in Stage 2 of the three-stage matching market mechanism, we can now determine the expected optimal behaviour in the selection of trade partners by both types of agents to form a matching structure. The main insight is presented in the next theorem, which indicates that agents of both types prefer to be matched with a partner of the other type. This then results in a description of the resulting equilibrium matching structures in this economy.

Theorem 3.1 *Assume that* $1 \leqslant Q \leqslant 3$*. Given the possible trade relationships and the Nash equilibria emerging in the second stage of the matching network trading process, both Type A and Type B agents weakly prefer to to be matched with an agent of the opposite type.*

Proof. The proof of the assertion stated in Theorem 3.1 will be checked for agents of both types separately.

Type A agents: Comparing a Type A – Type A meeting with a Type A – Type B meeting we derive the utilitarian comparisons that show that Type A always weakly prefer to be matched with a Type B agent.

Indeed, if $0 \leqslant \alpha \leqslant \frac{1}{1+2\sqrt{2}}$ we note that for any Type A agent the expected payoff in a Type A – Type A meeting is at most $U_{AA}(A) = \frac{Q(1+3\alpha)^2}{4(1+2\alpha)}$, while the equilibrium payoff from a Type A – Type B meeting is $U_{AB}(A) = \frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}$. It is easy to check that $U_{AB}(A) \geqslant U_{AA}(A)$ since $Q \geqslant 1$.

For $\alpha \geqslant \frac{1}{1+2\sqrt{2}}$ we note that $U_{AA}(A) = \alpha Q$ and that the payoff from a meeting with a Type B agent is either $U_{AB}(A) = \frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}$ or $U_{AB}(A) = \alpha Q$. In either case it is clear that $U_{AB}(A) \geqslant U_{AA}(A)$.

Type B agents: We first consider $0 \leqslant \alpha \leqslant \frac{Q}{1+2\sqrt{2}}$. In that case we have that the minimal payoff from a meeting with a Type B agent is $U_{BB}(B) = \frac{(Q+\alpha)^2}{4(Q+2\alpha)}$ and $U_{AB}(B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$. Clearly, $U_{AB}(B) \geqslant U_{BB}(B)$. So, even though the maximal payoff from a meeting with another Type B agent is larger, the fact that the minimum is attained implies that Type B agents weakly prefer to meet with Type A agents.

Next, for $\frac{Q}{1+2\sqrt{2}} \leqslant \alpha \leqslant Q$, we identify that the payoff from a meeting with a Type B agent is $U_{BB}(B) = \alpha$. For a meeting with a Type A agent we have that, depending on the exact value of α, either $U_{AB}(B) = \alpha$ (in the parameter subspace $\mathbb{B}$) or $U_{AB}(B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$ (in the parameter subspace $\mathbb{A}$). In both cases, a Type B agent weakly prefers a miring with a Type A agent.

Finally, for $\alpha \geqslant Q$, since $1 \leqslant Q \leqslant 3$, we have that $U_{AB}(B) = U_{BB}(B) = \alpha$, which implies that a Type B agent is indifferent between meeting with an agent of any type.

This completes the proof of Theorem 3.1. ∎

The analysis presented in the proof of Theorem 3.1 is summarised in Figure 2 below. In this graphical representation, the relevant parameter space is divided in the subspaces identified in the proof above.

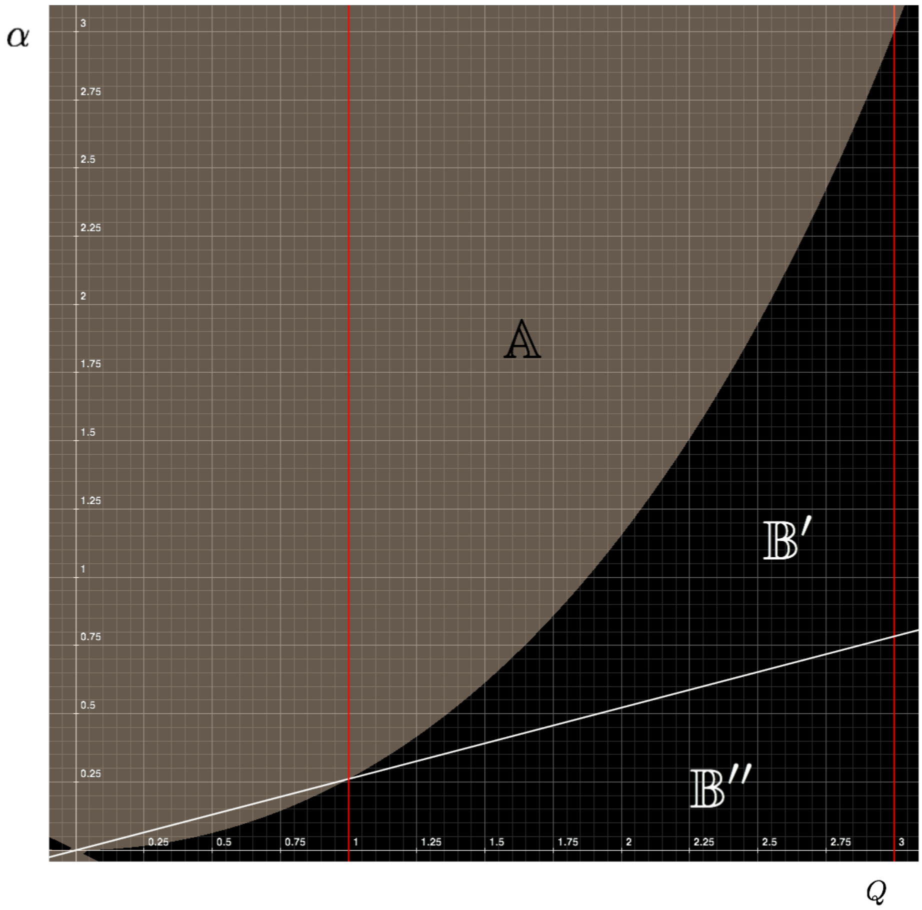

Figure 2: Equilibrium analysis for the matching economy

In Figure 2, the white line dividing $\mathbb{B}'$ from $\mathbb{B}''$ is described by the equation $\alpha = \frac{Q}{1+2\sqrt{2}}$. Now, in subspace $\mathbb{A}$ the prevailing equilibrium is a state of farming autarky (X, X), while in subspaces $\mathbb{B}'$ and $\mathbb{B}''$ the prevailing equilibrium is a developed social division of labour described by (X, Y). In the subspace $\mathbb{B}''$ all Type B agents have a preference to obtain a high payoff in a Type B – Type B meeting, but are deterred by the possibility that they actually get the minimal payoff from such a meeting.

From this analysis we can now give a precise and full description of the resulting equilibrium configuration of the matching network economy.

Corollary 3.2 *In the three-stage trade mechanism described by the matching network economy, there emerges a trade structure based on meetings asa follows:*

(i) *If* $0 < f \leqslant \frac{1}{2}$*, there emerges an equilibrium trade infrastructure with a fraction* $2f$ *of A–B meetings and a fraction* $(1-2f)$ *of B–B meetings.*

(ii) *If* $\frac{1}{2} \leqslant f < 1$*, there emerges an equilibrium trade infrastructure with a fraction* $2(1-f)$ *of A–B meetings and a fraction* $(2f-1)$ *of A–A meetings.*

We can summarise the payoff structures achieved in the various parameter subspaces as follows:

Type A – Type A meeting:

$$U = (\,\alpha Q\,,\,\alpha Q\,) \qquad \textit{for } \alpha > \tfrac{1}{1+2\sqrt{2}}$$

$$U = \left(\frac{Q(1+3\alpha)^2}{4(1+2\alpha)}\,,\,\frac{Q(1+\alpha)^2}{4(1+2\alpha)}\right) \qquad \textit{for } 0 \leqslant \alpha \leqslant \tfrac{1}{1+2\sqrt{2}}$$

Type A – Type B meeting:

$$U = (\,\alpha Q\,,\,\alpha\,) \qquad \textit{for subspace } \mathbb{A}$$

$$U = \left(\frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}\,,\,\frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}\right) \qquad \textit{for subspace } \mathbb{B}$$

Type B – Type B meeting:

$$U = (\,\alpha\,,\,\alpha\,) \qquad \textit{for subspace } \mathbb{A} \cup \mathbb{B}'$$

$$U = \left(\frac{(Q+3\alpha)^2}{4(Q+2\alpha)}\,,\,\frac{(Q+\alpha)^2}{4(Q+2\alpha)}\right) \qquad \textit{for subspace } \mathbb{B}''$$

4 A Yangian global market economy

The global market is described through a system of two markets in which agents submit their production bundles and in which a global market price for both goods is established. As usual, consumption plans are obtained through utility maximisation given the income generated by the trade of the produced quantities of the two goods. In the Yangian framework, given the prevailing price in these markets, all consumer-producers select a production plan from their production set. In this case, all agents either specialise in farming or in weaving. This results in an endogenous emergence of a social division of labour in the economy that is guided through the market price (Yang, 2001, Chapter 2).

As before, the price of cloth (Y) is expressed as in exchange rate in terms of the quantity of food stuff (X) required to obtain one unit of cloth. The market price of cloth is denoted in general by p, while the equilibrium market price under various conditions is represented as P. There emerge three potential equilibria in this Yangian general equilibrium framework with endogenous production planning by the two types of agents:

Price equilibration: All types of agents select their optimal production plan, being $(Q, 0)$ for Type A agents and $(0, Q)$ for Type B agents, resulting in a social division of labour that assigns both types of agents to their maximal productivity level. The market price now equilibrates demand and supply of the two goods in accordance with the standard Walrasian model with fixed production/supply. This type of price equilibration only exists and functions if and only if $\frac{1}{Q} < p < Q$.

Type A equilibration: If $p = Q$ then Type A agents become indifferent between farming and weaving. This implies there is potentially an equilibrium in this market economy at which

$p = Q$ and a fraction of weaving Type A agents equilibrates the markets. Thus, equilibration is accomplished through adjustment of the social division of labour.

Type B equilibration: Finally, if $p = \frac{1}{Q}$, then Type B agents become indifferent between weaving and farming. In that case there is potentially an equilibrium in this market economy at which $p = \frac{1}{Q}$ and a fraction of farming Type B agents equilibrates the markets. Thus, as in the previous case, equilibration is accomplished through adjustment of the social division of labour.

The Yangian framework based on the modelling of economic agents as consumer-producers results in a very different outlook than a classical Walrasian approach based on a dichotomy of production and consumption. Indeed, the indigenous adaptation of the social division of labour results in the emergence of equilibria that are not based on price equilibration, but equilibration through the adaptation of the social division of labour. In particular, this refers here to the presence of Type A and Type B equilibration in the global market economy.

4.1 Price equilibration

The market price p is adjusted so that demand and supply equalise for a given population of X- and Y-producers. The condition is that

$$\frac{1}{Q} < p < Q \tag{28}$$

In that case all agents specialise in the task they accomplish in the most productive fashion; i.e., all agents strive for income maximisation. That means that Type A agents farm and produce Q units of X and that Type B agents weave and produce Q units of Y. Thus, the market economy is described by demand and supply given by

$$D_X(p) = f \cdot \frac{Q + \alpha p}{2} + (1 - f) \cdot \frac{(Q + \alpha)p}{2} \tag{29}$$

$$D_Y(p) = f \cdot \frac{Q - \alpha p}{2p} + (1 - f) \cdot \frac{Q - \alpha}{2} \tag{30}$$

$$S_X(p) = f \cdot Q \tag{31}$$

$$S_Y(p) = (1 - f) \cdot Q \tag{32}$$

Market equilibration in X implies that from $D_X(p) = S_X(p)$ the equilibrium price is determined as

$$P^* = \frac{fQ}{(1 - f)Q + \alpha} \tag{33}$$

Consumption bundles in equilibrium for both type of agents can now be determined as

$$C^A = \left(\frac{(1-f)Q^2 + \alpha(1-+f)Q}{2(1-f)Q + 2\alpha}, \frac{(1-f)(Q+\alpha)}{2f} \right) \tag{34}$$

$$C^B = \left(\frac{fQ(Q+\alpha)}{2(1-f)Q + 2\alpha}, \frac{Q-\alpha}{2} \right) \tag{35}$$

Thus, the equilibrium utility levels through price equilibration are now computed as

$$U^A = \frac{Q\left((1-f)Q + \alpha(1+f) \right)^2}{4f(1-f)Q + 4\alpha f} \tag{36}$$

$$U^B = \frac{fQ\left(Q+\alpha \right)^2}{4(1-f)Q + 4\alpha} \tag{37}$$

Now the fundamental price equilibration condition (28) holds if and only if

$$(Q+1)f - Q < \alpha < fQ(Q+1) - Q \tag{38}$$

which is equivalent to the condition that

$$\frac{Q+\alpha}{Q(Q+1)} < f < \frac{Q+\alpha}{Q+1} \tag{39}$$

4.2 Type A equilibration

This case is described by equilibration of the market economy through the adjustment of the social division of labour for the fixed market price of cloth given by $P_A = Q$.

In this case we know that incomes for both types are given by $I_A = Q$ and $I_B = Q^2$. At the specified price, Type A agents are indifferent between farming and weaving, while Type B agents are always weaving. Thus, we can assume that there is a fraction $0 \leqslant g \leqslant f$ of Type A agents that specialise in weaving rather than farming, while a fraction $f - g$ of Type A agents specialises as farmers.

The market economy is now described by total market demand and supply for X as well as Y given by

$$D_X(P_A) = f \cdot \frac{1+\alpha}{2}Q + (1-f) \cdot \frac{Q+\alpha}{2}Q \tag{40}$$

$$D_Y(P_A) = f \cdot \frac{1-\alpha}{2} + (1-f) \cdot \frac{Q-\alpha}{2} \tag{41}$$

$$S_X(P_A) = f \cdot Q + g \cdot 1 \tag{42}$$

$$S_Y(P_A) = (1-f-g) \cdot Q \tag{43}$$

Through market equilibration for X we can now easily compute that in equilibrium

$$\hat{g} = \frac{f(Q+1) - (Q+\alpha)}{2} \tag{44}$$

It is easy to compute from individual demand functions that equilibrium utility levels are now given by

$$U^A = \frac{Q(1+\alpha)^2}{4} \tag{45}$$

$$U^B = \frac{Q(Q+\alpha)^2}{4} \tag{46}$$

This equilibrium exists if $0 \leqslant \hat{g} \leqslant f$, which is equivalent to the condition that

$$(Q-1)f - Q \leqslant \alpha \leqslant (Q+1)f - Q \tag{47}$$

This in turn can be expressed as

$$\frac{Q+\alpha}{Q+1} \leqslant f \leqslant \frac{Q+\alpha}{Q-1} \tag{48}$$

4.3 Type B equilibration

In this case, the market price of cloth is fixed at $P_B = \frac{1}{Q}$. Again market equilibration is achieved through the adjustment of the social division of labour.

In this case we know that incomes for both types are given by $I_A = Q$ and $I_B = 1$. At the specified price, Type A agents always farm, while Type B agents are indifferent between farming and weaving. Thus, we can assume that there is a fraction $0 \leqslant g \leqslant 1 - f$ of Type B agents that specialise in farming rather than weaving, while a fraction $1 - f - g$ specialises as weavers.

The market economy is now described by total market demand and supply given by

$$D_X(P_B) = f \cdot \frac{Q^2+\alpha}{2Q} + (1-f) \cdot \frac{Q+\alpha}{2Q} \tag{49}$$

$$D_Y(P_B) = f \cdot \frac{Q^2-\alpha}{2} + (1-f) \cdot \frac{Q-\alpha}{2} \tag{50}$$

$$S_X(P_B) = f \cdot Q + g \cdot 1 \tag{51}$$

$$S_Y(P_B) = (1-f-g) \cdot Q \tag{52}$$

Through market equilibration for Y we can now determine that in equilibrium

$$\hat{g} = \tfrac{1-f}{2} - \tfrac{1}{2}f + \tfrac{\alpha}{2Q} \tag{53}$$

It is easy to compute from individual demand functions that equilibrium utility levels are now given by

$$U^A = \frac{(Q^2+\alpha)^2}{4Q} \tag{54}$$

$$U^B = \frac{(Q+\alpha)^2}{4Q} \tag{55}$$

This equilibrium exists if $0 \leqslant \hat{g} \leqslant 1 - f$. This holds if and only if

$$fQ(Q+1) - Q \leqslant \alpha \leqslant fQ(Q-1) + Q \tag{56}$$

This is, in turn, equivalent to

$$\frac{\alpha - Q}{Q(Q-1)} \leqslant f \leqslant \frac{\alpha + Q}{Q(Q+1)} \tag{57}$$

5 Inequality analysis

We are now in the position to analyse the changes to utilitarian inequality in the economy under different institutional frameworks. So, for given populations we can analyse how a matching network economy compares to a global market economy in terms of the generated utility values.

We consider three types of populations, namely a population with a restricted proportion of Type A agents; a balanced population with equal fractions of A and B types; and a population with a larger proportion of Type A agents. For each of these cases we compare trade outcomes under the two institutional structures we have examined in the previous sections of this paper.

Throughout this analysis we assume that $1 \leqslant Q \leqslant 3$. Furthermore, we emphasise that this analysis only has validity in the parameter subspace $\mathbb{B}$. First, for any fraction $f \in [0,1]$ it is clear that if $\alpha > Q$ the economy collapses in autarky, regardless of the institutional trade structure considered. Indeed, also in a global market Type B agents are not willing to become weavers under these parameter values. Second, for the subspace $\mathbb{A}$ with $\alpha \leqslant Q$ the matching network economy collapses in a farming autarky. Hence throughout the parameter subspace $\mathbb{A}$, the analysis would not be very fruitful.

This leaves the following cases for consideration:

Case I: $f = \frac{1}{4}$ and $(Q, \alpha) \in \mathbb{B}''$.

This describes the case of a small, privileged class of Type A agents such that trade is very relevant, also in the matching framework.

Case II: $f = \frac{1}{4}$ and $(Q, \alpha) \in \mathbb{B}'$.

This describes the case of a small, privileged class of Type A agents such that trade is only relevant in a global market, but autarky becomes relevant for Type B – Type B relationships in the matching framework.

Case III: $f = \frac{1}{2}$ and $(Q, \alpha) \in \mathbb{B}$.

This describes the case of a completely balanced population of economic agents. In this case the global market performs exactly the same under price equilibration as the matching framework. However, a secondary Type B equilibrium in the global market still imposes inequality differences under these two institutional settings.

Case IV: $f = \frac{3}{4}$ and $0 \leqslant \alpha \leqslant \frac{1}{1+2\sqrt{2}}$.

If there is a minority of Type B agents, there emerges interesting consequences for the inequality in the economy. In this case, Type A agents suffer welfare losses from globalisation.

On the other hand, Type B agents gain from globalisation. The consequences for inequality are unpredictable, including the case of inequality reversal.

Case V: $f = \frac{3}{4}$ and $\alpha \geqslant \frac{1}{1+2\sqrt{2}}$.

In this final case, the analysis is less straightforward. For certain parameter values, where Q and α are both relatively high, there emerge strict Pareto improvements under globalisation.

Each of these five cases is discussed next.

5.1 Analysis for Case I

First, consider $f = \frac{1}{4}$ and $(Q, \alpha) \in \mathbb{B}''$. Hence, Type A agents form a minority, which enhances their scope as specialist producers of the socially preferred commodity X. In particular we look at a fraction $f = \frac{1}{4}$ of Type A agents, implying that there is a fraction $1 - f = \frac{3}{4}$ of Type B agents.

We are particularly interested in finding evidence that global trade results in larger inequalities in the economy. This implies that we focus on the parameter subspace $\mathbb{B}''$ for which Type B agents achieve a relatively high payoff in trade with other Type B agents. This will, therefore, be the focus of our investigation for the case that there are more Type A agents than Type B agents and Type B – Type B matchings are common under the matching network institutional arrangement.

The matching economy: We consider the different equilibria that might arise from the matching structure in the economy. In particular, we know that all Type A agents match with a Type B agent and that the remaining Type B agents match among themselves. We assume that matching is essentially random within these pairings, so any Type B agent has equal probability to assume any of the three equilibrium roles in these matchings.[7]

In the parameter subspace $\mathbb{B}''$ we get the following equilibrium utilities:

$$U_{AB}(F_A) = \frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}$$

$$U_{AB}(W_B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$$

$$U_{BB}(F_B) = \frac{(Q+3\alpha)^2}{4(Q+2\alpha)}$$

$$U_{BB}(W_B) = \frac{(Q+\alpha)^2}{4(Q+2\alpha)}$$

Hence, we conclude that the expected utility levels for the two agent types in this matching econ-

[7]Hence, any Type B agent specialises with a probability $\frac{1}{3}$ as a weaver engaging with a Type A agent; as a farmer engaging with another Type B agent; and as a weaver engaging with another Type B agent.

omy are given by

$$U_N(A) = \frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)} \tag{58}$$

$$U_N(B) = \tfrac{1}{3}U_{AB}(W_B) + \tfrac{1}{3}U_{BB}(F_B) + \tfrac{1}{3}U_{BB}(W_B) = \frac{(Q+1)(Q+\alpha)^2 + (Q+3\alpha)^2}{12(Q+2\alpha)} \tag{59}$$

These utility values can now be compared to the utility levels achieved under a global market mechanism.

The global market economy: Again we limit our focus to the parameter subspace $\mathbb{B}''$ in our analysis of the global market system. Now, given the parameter inequalities given in (38), (47) and (56) for the stated value $f = \frac{1}{4}$, we identify that for $1 \leqslant Q \leqslant 3$ there are no values of α in which price equilibration or Type A equilibration functions. The only global market equilibrium is therefore based on Type B equilibration. In the resulting equilibrium the price is given by $P = \frac{1}{Q}$ and a fraction $\hat{g} = \frac{1}{4} + \frac{\alpha}{2Q} \leqslant \frac{3}{4} = 1 - f$ of Type B agents specialise as farmers.

For this form of market equilibration we derive that the expected utility values are given by

$$U_G(A) = \frac{(Q^2+\alpha)^2}{4Q} \tag{60}$$

$$U_G(B) = \frac{(Q+\alpha)^2}{4Q} \tag{61}$$

Inequality analysis: Using the identified utility values for the different institutional systems, we can now do a complete comparison. In particular, we are interested whether the majority Type B agents benefit from globalisation. We therefore consider the inequality

$$U_N(B) > U_G(B).$$

A secondary comparison of interest is the one given by

$$U_N(A) > U_G(A).$$

Figure 3 depicts the graphical outcome of this comparative analysis. The *orange* area intersected with the subspace $\mathbb{B}''$ indicates the parameter values for which all Type B agents are not expected to benefit from globalisation. This mainly refers to the area in $\mathbb{B}''$ at which α and Q levels are higher. In this area, all Type A agents benefit from globalisation, thereby increasing inequality between Type A and Type B agents in the economy.

In the (dark) *gray* area in $\mathbb{B}''$ to the left, where Q is closer to the lower bound of $Q = 1$, Type A agents suffer a welfare loss from globalisation, while Type B agents gain from globalisation. This implies that in the gray area the welfare inequality diminishes.

Only in the very small *black* area in $\mathbb{B}''$—between the gray and orange areas—there is actually a strict Pareto improvement for both types of agents from globalisation.

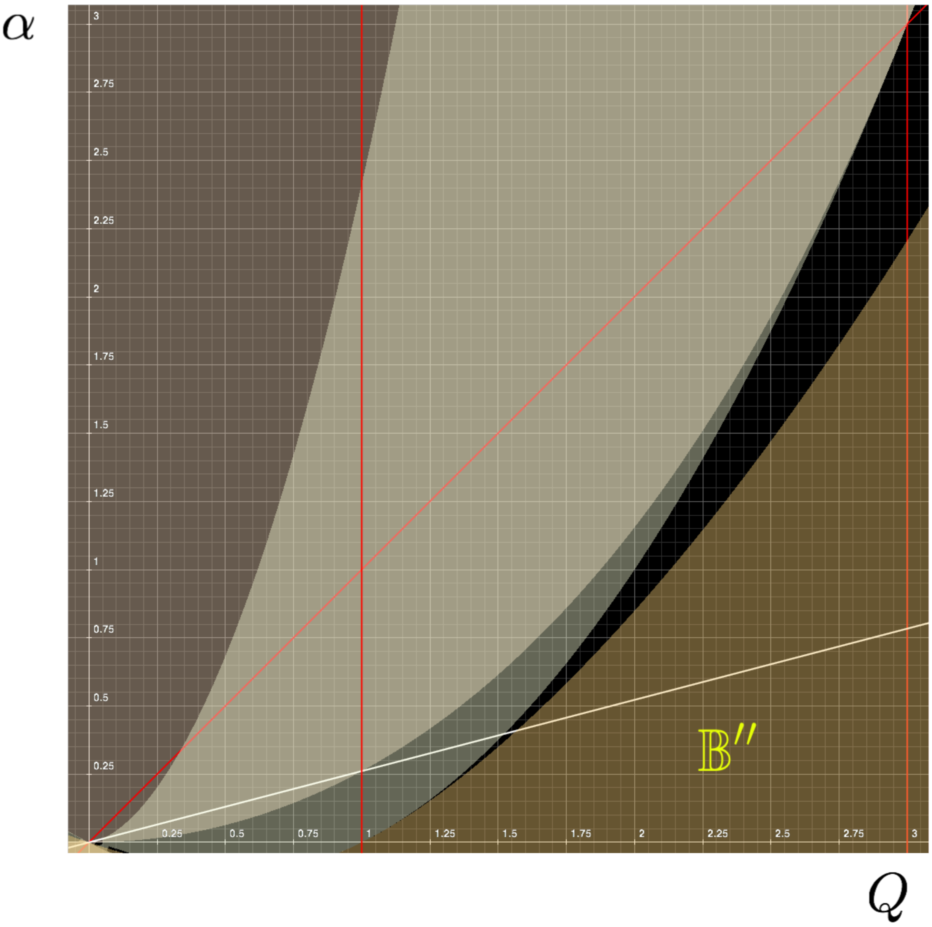

Figure 3: Inequality analysis for Case I.

We conclude that, within the subspace $\mathbb{B}''$, there are three different areas of interest with regard to inequality between Type A and Type B agents: In the gray area inequality decreases from globalisation; in the black area there is a strict Pareto improvement from globalisation under which inequality remains relatively unchanged; and in the orange area the majority suffers welfare losses from globalisation and there is an inequality increase.

5.2 Analysis for Case II

Next, consider again a low fraction of Type B agents with $f = \frac{1}{4}$ and $(Q, \alpha) \in \mathbb{B}'$. Due to the restriction of the parameters $(Q, \alpha) \in \mathbb{B}'$ in the matching economy we now get that

$$U_{BB}(F_B) = U_{BB}(W_B) = \alpha. \tag{62}$$

Hence, $U_N(A)$ remains unchanged from Case I, while

$$U_N(B) = \frac{Q(Q+\alpha)^2}{12(Q+2\alpha)} + \frac{2\alpha}{3} \tag{63}$$

with regard to a global market, the only viable equilibration remains that of Type B, thus resulting are unchanged from the ones reported in Case I.

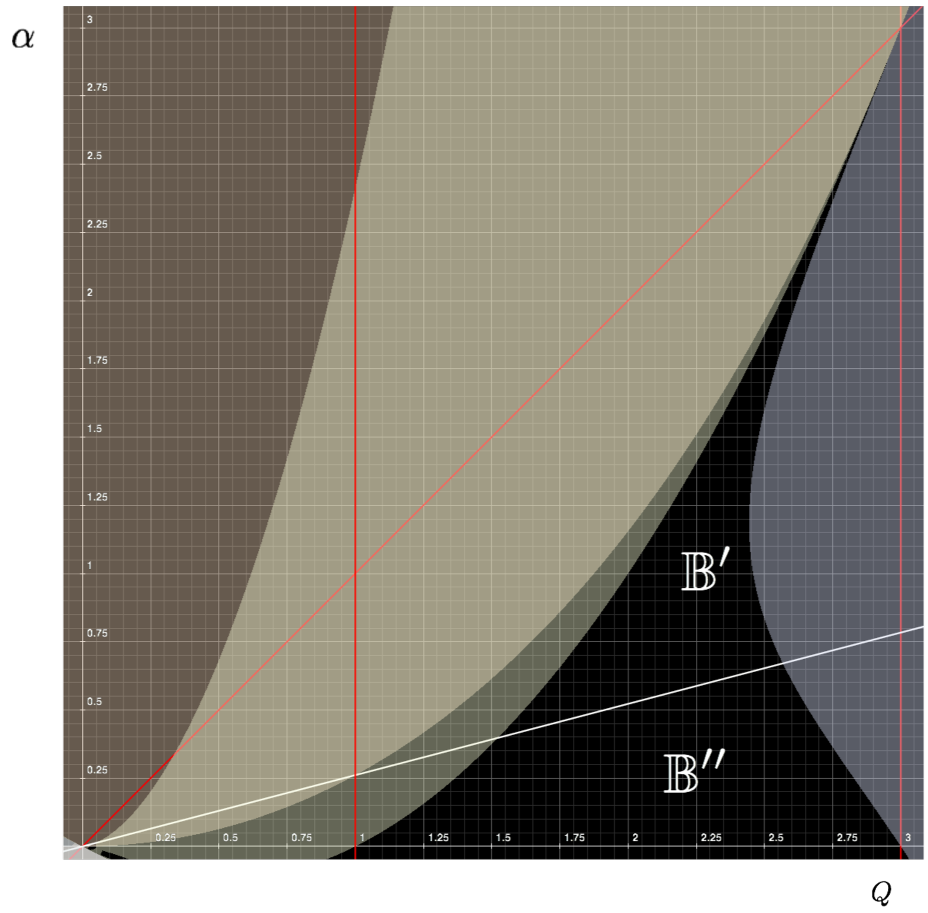

Figure 4: Inequality analysis for Case II.

Welfare comparisons result in the graph depicted in Figure 4. Restricting our attention to the parameter subspace $\mathbb{B}'$, we see three pronounced disjoint areas of interest: In the gray area to the left, Type A agents have welfare losses from globalisation; in the black area in the middle, there is a strict Pareto improvement from globalisation; and in the blue area to the left Type B agents suffer welfare losses due to globalisation.[8]

It is clear that welfare losses for Type B agents are much less widespread due to the low return from trade relationships with other Type B agents in the matching economy. Nevertheless, for Q sufficiently high, globalisation results in inequality increases.

5.3 Analysis of a balanced population: Case III

Next, consider $f = \frac{1}{2}$ and $(Q, \alpha) \in \mathbb{B}$. Here the population is completely balanced between Type A and Type B agents. This balance simplifies the analysis considerably for the matching economy

[8]It is emphasised that the analysis is only valid in and refers to the parameter subspace $\mathbb{B}'$, excluding the subspace $\mathbb{B}''$ from consideration.

as well as for the global market under price equilibration. Again we investigate the equilibria that emerge under the two institutional frameworks and analyse the implications for inequality.

The matching economy: In the matching framework, there only emerge Type A – Type B relationships due to the population balance between the two agent types. Hence, the equilibrium payoffs are simply the payoffs computed for this relationship form:

$$U_N(A) = \frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)}$$

$$U_N(B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$$

The global market economy: In the global economy, there emerge two different type of equilibria. With reference to the depiction in Figure 5, we can distinguish two areas in the subspace $\mathbb{B}$, separated by the green curve described by the equation

$$\alpha = \tfrac{1}{2}Q(Q-1). \tag{64}$$

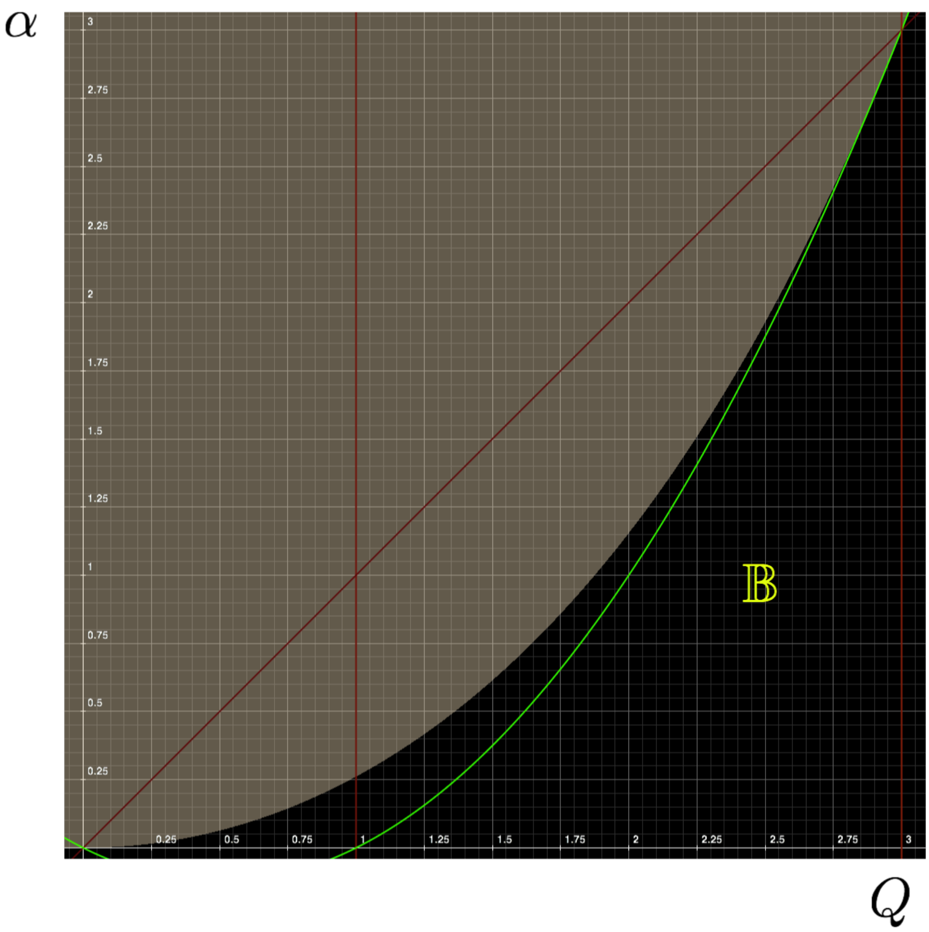

Figure 5: Inequality analysis for Case III.

The area to the left of the green curve represents the area in $\mathbb{B}$ in which price equilibration emerges in the global market, while in the parameter set right of the green curve there emerges a Type B equilibrium in the global market.

Right to the green curve ($\alpha \leqslant \frac{1}{2}Q(Q-1)$), the equilibrium utilities from price equilibration can be computed as

$$U_G(A) = \frac{Q(Q+3\alpha)^2}{4(Q+2\alpha)} \equiv U_N(A) \tag{65}$$

$$U_G(B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)} \equiv U_N(B) \tag{66}$$

Hence, as expected under a balanced population, price equilibration results in exactly the same payoffs as one-to-one matching between agents of the two different types.

Left to the green curve ($\alpha \geqslant \frac{1}{2}Q(Q-1)$), the equilibrium utilities from Type B equilibration can be computed as

$$U_G(A) = \frac{(Q^2+\alpha)^2}{4Q} \tag{67}$$

$$U_G(B) = \frac{(Q+\alpha)^2}{4Q} \tag{68}$$

This leads to the question whether there are welfare inequalities in the area of the parameter subspace $\mathbb{B}$ that support Type B equilibration. It is easy to check that $U_N(A) > U_G(A)$ as well as $U_N(B) < U_G(B)$ for the area left to the green curve in Figure 5. Hence, Type B agents gain from globalisation, while Type A agents suffer utilitarian losses from globalisation.

As a consequences the welfare inequality between Type A and Type B agents will diminish by globalisation through the transforming the institutional economic framework to a global market.

5.4 Analysis of Case IV

Now consider that Type A agents form a majority, say $f = \frac{3}{4}$. Our previous analysis shows that we have distinguish $\alpha \leqslant \frac{1}{1+2\sqrt{2}}$ (Case IV) from $\alpha \geqslant \frac{1}{1+2\sqrt{2}}$ and $(Q,\alpha) \in \mathbb{B}$ (Case V). (Note here that in Case IV it automatically holds that $(Q,\alpha) \in \mathbb{B}$.)

The matching economy: If Type A agents form a majority, in a matching economy they assume with equal probability the three roles of farmer in a Type A – Type B and the two different roles in a Type B – Type B relationship. Hence, the expected utility for any Type A agent in the matching economy can be computed as

$$\begin{aligned} U_N(A) &= \tfrac{1}{3}U_{AB}(F_A) + \tfrac{1}{3}U_{AA}(F_A) + \tfrac{1}{3}U_{AA}(W_A) = \\ &= \frac{Q(Q+3\alpha)^2}{12(Q+2\alpha)} + \frac{Q(1+4\alpha+5\alpha^2}{6(1+2\alpha)} \end{aligned} \tag{69}$$

Also we derive for all Type B agents that their utility in the matching economy is given by

$$U_N(B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)}$$

The global market economy: In Case IV, there emerge two forms of equilibration in the market: Price equilibration and Type A equilibration, depending on the exact parameter values for the identified area. In both cases the analysis leads to exactly the same conclusions.[9]

Indeed, under price equilibration we arrive at utilities given by

$$U_G(A) = \frac{Q(Q+7\alpha)^2}{12(Q+4\alpha)} \tag{70}$$

$$U_G(B) = \frac{3Q(Q+\alpha)^2}{4(Q+4\alpha)} \tag{71}$$

In this case it holds for all relevant parameter values that $U_N(A) > U_G(A)$ as well as $U_N(B) < U_G(B)$.

Similarly, under Type A equilibration we derive that

$$U_G(A) = \frac{Q(1+\alpha)^2}{4} \tag{72}$$

$$U_G(B) = \frac{Q(Q+\alpha)^2}{4} \tag{73}$$

Again in the relevant parameter space, it holds that $U_N(A) > U_G(A)$ as well as $U_N(B) < U_G(B)$.

Hence, uniformly one can conclude that globalisation leads to welfare losses for Type A agents, while Type B agents gain from globalisation. Regarding inequality, this implies that inequality diminishes or even reverses and in some cases that inequality in favour of Type B agents increases further.

5.5 Analysis of Case V

Finally, consider the case that $f = \frac{3}{4}$, $\alpha \geqslant \frac{1}{1+2\sqrt{2}}$ and $(Q, \alpha) \in \mathbb{B}$. In this case for the matching economy we derive

$$U_N(A) = \frac{2\alpha Q}{3} + \frac{Q(Q+3\alpha)^2}{12(Q+2\alpha)} \tag{74}$$

$$U_N(B) = \frac{Q(Q+\alpha)^2}{4(Q+2\alpha)} \tag{75}$$

In the global market there are two valid equilibration forms: In Figure 6, the green line represented by $4\alpha = 3 - Q$ delineates these two cases. Right to this line, the parameters support price equilibration, while left to this green line the parameter values lead to Type A equilibration in the global market.

[9] The exact parameter values are depicted in Figure 6. The green line is represented by $4\alpha = 3 - Q$, below the red line with $\alpha = 1/(1 + 2\sqrt{2})$. For all parameter values right to this line there emerges price equilibration, while for all parameter values left to this line there emerges Type A equilibration in the global market.

Under price equilibration, we derive utility values given by

$$U_G(A) = \frac{Q(Q + 7\alpha)^2}{12(Q + 4\alpha)}$$

$$U_G(B) = \frac{3Q(Q + \alpha)^2}{4(Q + 4\alpha)}$$

as already computed for Case IV above.

Again the conclusion is that under price equilibration Type A agents suffer losses, while Type B agents gain from globalisation as already reported in Case IV.

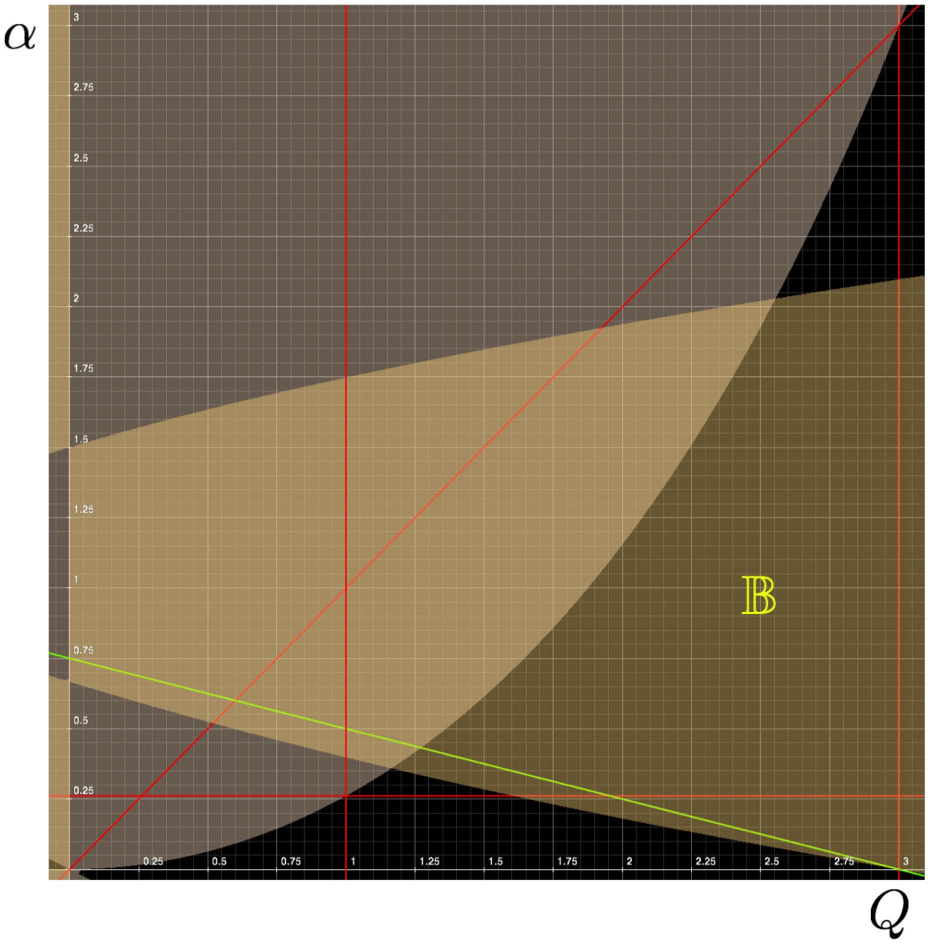

Figure 6: Inequality analysis for Case V.

Under Type A equilibration in the global market there emerge the same utility levels already reported under Case IV. Now, however, a utilitarian comparison results in more intricate conclusions. As before all Type B agents gain from globalisation: $U_N(B) < U_G(B)$. On the other hand, it holds that $U_N(A) > U_G(A)$ in the orange area in Figure 6, left/below the green line. In the black area towards the corner, the reverse holds, and Type A gain from globalisation.

5.6 Overview of the analytical conclusions

In this paper we looked in depth at the parameter values in which inequalities between the two types of economic agents increase due to globalisation. Contemplating and reviewing the results presented above for all Cases I–V, we conclude that the following general properties hold:

Theorem 5.1 *In the Yangian general equilibrium framework with two types of agents considered here, we conclude that*

(i) *Type A agents are always better off than Type B agents, irrespective of the institutional trade infrastructure imposed in this economy;*

(ii) *there emerge more trade opportunities and more fully developed equilibrium configurations in the matching economy for cases in which the necessary good (X) is more equally preferred to the luxury good (Y), represented by lower values of α, while for high enough values of α there emerges a farming autarky independently of the trade institutional framework;*

(iii) *for higher returns to specialisation—represented by higher values of Q—inequality between Type A and Type B agents increases under globalisation;*

(iv) *for medium values of both α and Q globalisation is uniformly welfare increasing, leading to strict Pareto improvements across all types of economic agents;*

(v) *and for lower returns to specialisation—represented by values of Q closer to unity (1)—inequality between Type A and Type B agents decreases due to globalisation.*

It is clear that further research is necessary to clarify the questions about the emergence of certain equilibria, the effects of globalisation on autarky versus trade, and increasing inequality due to globalisation.

References

Cheng, W., and X. Yang (2004): "Inframarginal Analysis of Division of Labor: A Survey," *Journal of Economic Behavior and Organization*, 55, 137–174.

Debreu, G. (1951): "The Coefficient of Resource Utilization," *Econometrica*, 19, 273–292.

Diamantaras, D., and R. P. Gilles (2004): "On the Microeconomics of Specialization," *Journal of Economic Behavior and Organization*, 55, 223–236.

Foley, D. K. (1967): "Resource Allocation and the Public Sector," *Yale Economic Essays*, 7, 45–98.

Heckscher, E. (1919): "The Effect of Foreign Trade on the Distributrion of Income," *Ekonomisk Tidskrif*, 21, 1–32.

Kolm, S. (1972): *Justice et Equité*. Editions du Centre National de la Recherche Scientifique, Paris, France.

Ohlin, B. G. (1933): *Interregional and International Trade*. Harvard University Press, Cambridge, MA.

Pareto, V. (1906): *Manual of Political Economy*. Macmillan Press, London, UK, 1972 reprint edn.

Pazner, E. (1977): "Pitfalls in the Theory of Fairness," *Journal of Economic Theory*, 14, 458–466.

Pazner, E., and D. Schmeidler (1978): "Egalitarian Equivalent Allocations: A New Concept of Economic Equity," *Quarterly Journal of Economics*, 92, 671–687.

Piketty, T. (2014): *Capital in the Twenty-First Century*. Harvard University Press, Boston, MA.

Schmeidler, D., and K. Vind (1972): "Fair Net trades," *Econometrica*, 40, 637–664.

Stiglitz, J. E. (2013): *The Price of Inequality*. Penguin, London, UK.

Thomson, W. (1983): "Equity in Exchange Economies," *Journal of Economic Theory*, 29, 217–244.

Varian, H. A. (1974): "Equity, Envy, and Efficiency," *Journal of Economic Theory*, 9, 63–91.

Yang, X. (1988): "A Microeconomic Approach to Modeling the Division of Labor Based on Increasing Returns to Specialization," Ph.D. thesis, Princeton University, Princeton, New Jersey.

——— (2001): *Economics: New Classical Versus Neoclassical Frameworks*. Blackwell Publishing, Malden, Massachusetts.

——— (2003): *Economic Development and the Division of Labor*. Blackwell Publishing, Malden, Massachusetts.

Yang, X., and J. Borland (1991): "A Microeconomic Mechanism for Economic Growth," *Journal of Political Economy*, 99, 460–482.

Yao, S. (2002): "Walrasian Equilibrium Computation, Network Formation, and the Wen Theorem," *Review of Development Economics*, 6, 415–427.